THE DUSTON SCHOOL
Knowledge itself is power

Year 10 Maths Higher Knowledge Booklet Term 1

Name:

Class:

Order	Unit	Links	Pre-requisite skills
1	Integers, powers & roots		
2	Lines, angles & shape		
3	Simplifying & substituting	Unit 1	Using powers, listing factors, understanding product / sum.
4	Area and perimeter	Unit 2	Forming expressions for area/perimeter algebraically through use of brackets, correct notation and simplifying expressions.
5	Calculations & Accuracy	Unit 1	Understanding numbers.
6	Construction and LOCI	Unit 2	Measuring angles for bearings, parallel line angle facts.
7	FDP	Unit 1	Using powers, understanding lowest common multiples.
8	Sequences, functions and graphs	Unit 3/5	Substituting into a function applying BIDMAS to calculate coordinates, factorising for roots of quadratics, understanding powers and all 4 operations with negatives.
9	Ratio & Proportion	Unit 1/7	Decimals/powers as multipliers, calculating/understanding fractions as parts.
10	Transformations	Unit 2/8	Identifying 90/180/270 degrees, plotting mirror lines of basic functions.
11	Pythagoras and Trigonometry	Unit 1/2/3/4/5	Powers/surds, types of triangles, use in area/perimeter problems to find required lengths, rounding answers.
12	Forming and solving	Unit 3/4	Properties of 2d shapes, angle facts including polygons & parallel lines, algebraic notation and simplifying, forming expressions.
13	Measures	Unit 1/7	Calculating, multiplying decimals and powers of 10 for metric conversions.
14	Volume and Surface area	Unit 4/5/13	Area of 2d shapes, rounding/calculating with bounds, conversion of units (length/area/volume), calculating missing sides using pythagoras/ trigonometry.
15	Probability	Unit 1/7	Types of numbers, calculating with fractions & decimals.
16	Inequalities	Unit 12/8/5/7	Solving equations, rounding, plotting graphs for regions, calculating with fractions.
17	Statistics	Unit 1/6/9/16	Using a protractor for pie charts, proportion to calculate angles for a pie chart, use of inequality symbols for recording data.

Homework 1 Due	
Homework 2 Due	
Homework 3 Due	

Overview	**Learning Objective**		
Topic: Integers, Powers and Roots **Big Questions** Convince me that $\sqrt{72} = 6\sqrt{2}$ - What do I need to multiply $(4 + \sqrt{})$ by to give $11 + 6\sqrt{3}$ as a simplified answer?	- Calculate with standard index form. Multiplication. - calculate with standard form. Division. - Calculate with standard form. Addition. -Calculate in standard form. Substraction. - Use index notation for negative integer indices - Know that $n^{1/2} = \sqrt{n}$ and $n^{1/3} = \sqrt[3]{n}$ for any positive number n. - Use index notation and index laws for simple fractional powers such as $16^{3/4}$	- Use index notation involving fractional negative powers. - - Know that $n^{1/2} = \sqrt{n}$ and $n^{1/3} = \sqrt[3]{n}$ for any positive number n. - Use index notation and index laws for simple fractional powers such as $16^{3/4}$. - Simplify surds to the form $a\sqrt{b}$ -multiplying surds	- Use index notation involving fractional negative powers. - Rationalise a denominator in the form $a\sqrt{b}$, $a + \sqrt{b}$ and $a + b\sqrt{c}$. - Construct an algebraic proof of number properties. Simplify surds, such as $4(3 + \sqrt{3})$ and $(2 - \sqrt{3})(4 + \sqrt{3})$ in the form $a + b\sqrt{3}$ - Rationalise the denominator of a surd such as $2/\sqrt{5}$.
Topic: Lines, angles and shapes **Big Questions** - Show me a problem with a right-angle in a semi-circle. - Show me a problem where two angles are subtended by the same arc. - Show me a problem involving a cyclic quadrilateral.	- Use the tangent/radius properties of a circle. - Apply circle theorems. - Prove circle theorems.		
Topic: Simplifying and substituting **Big Questions** - Can you have a negative square root? - Expand $(a + b)^3$	- Expand products of more than two binomials. - Recognise and factorise the difference of two squares.	- Factorise quadratics in the form $ax^2 + bx + c = 0$ where $a = 1$. - Factorise quadratics in the form $ax^2 + bx + c = 0$ where $a > 1$.	- Simplify algebraic fractions that involve factorising. - Interpret the succession of two functions as a 'composite function'. - Interpret the reverse process as the 'inverse function'.

INTEGERS, POWERS & ROOTS KNOWLEDGE ORGANISER

Prime Number	A positive integer greater than 1 that can only be divided by itself and 1 without leaving a remainder.	7, 23, and 67 are examples. 1 is **not** a prime number.
Base	The big number on the left; the number that will be multiplied by itself.	5 is the base in 5^3
Index	The small number on the top-right; the number of times a value is multiplied by itself.	3 is the index in 5^3
Power of exponent	Another word for index.	3 to the power of 2 = 3^2
Indices	Plural of index	
Reciprocal	The reciprocal of a number is one divided by that number	The reciprocal of 4 is $\frac{1}{4}$ which is equal to 0.25
Steps to estimate a square root	1: Identify the **square on either side**. 2: Place the squares and their roots at **eit end of a blank line.** 3: Identify the **halfway** point. 4: **Place** your number on the line. 5: **Estimate** its position. 6: **Square** your answer to check.	

Integers

"Integer" is just a posh word for whole number.
The thing to remember is that integers can be positive or negative
So: 1, 7, 298, -3, 0 and -49 are all integers, but 2.5 is not and neither is $3\frac{5}{8}$!

Multiples

The Multiples of a number are all the numbers in your number's times table.
Don't forget: you must count the number itself!
e.g. Some multiples of 7 are: 7, 14, 21, 28... but there are loads more, like 700 and 4445

Factors

The Factors of a number are all integers that divide into your number exactly (there must not be a remainder!)
Don't forget: 1 is a factor of all numbers, and so is the number itself!
e.g. The factors of 12 are: 1, 2, 3, 4, 6 and 12

In index form	**Written using powers**	In index form, 4x4x4 is 4^3
Expand	Write in a **longer form, without indices**	7^3 expands to give 7x7x7
Evaluate	**Find the value**; work out	I would 2^5 to get 32
Simplify	Write in the **simplest**, neatest **form**	$2^3 \times 2^5$ simplifies to 2^8

Prime Factor Tree

Hint: Circle the prime numbers

$216 = 2x2x2x3x3x3$
$= 2^3x3^3$

Standard Form

A number in standard form must be written in this way.

A number between 1 and 10 × A power of 10

2×10^{15} and 1.5×10^{-12}

Going from standard form to ordinary numbers

$5 \times 10^{10} =$ 50 000 000 000 $\quad$ 0.0006 = 6×10^{-4}

$7.1 \times 10^{6} =$ 7 100 000 $\quad$ 0.00000072 = 7.2×10^{-7}

Look at the power: positive the number will get bigger, if negative the number will be smaller.

Multiplying Standard Form

$(a \times 10^n)(b \times 10^m) = (a \times b)(10^n \times 10^m)$

$(2\times10^6)\times(4\times10^4) = 2\times4\times10^6\times10^4$
$= 8\times10^{6+4}$

Dividing Standard Form

$\frac{a\times10^n}{a\times10^m} = \frac{a}{b} \times \frac{10^n}{10^m}$

$(8\times10^{13})\div(2\times10^7) = (8\div2)\times(10^{13}\div10^7)$
$= 4\times10^{13-7}$

Adding and Subtracting Standard Form

Step 1: Take the numbers in Standard Form back to an ordinary number.

Step 2: Add or subtract the numbers $\quad$ Step 3: Put the answer back into Standard Form

Surds

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
3. $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$
4. $0 = \sqrt{a} - \sqrt{a}$

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^n \times x^m = x^{n+m}$	$x^2 \times x^3 = x^{2+3} = x^5$
4	$x^n \div x^m = x^{n-m}$	$x^5 \div x^2 = x^{5-2} = x^3$
5	$(x^n)^m = x^{n\times m}$	$(x^3)^2 = x^{3\times2} = x^6$
6	$(x\times y)^n = x^n \times y^n$	$(x\times y)^3 = x^3 \times y^3$
7	$(x\div y)^n = x^n / y^n$	$(x\div y)^3 = x^3 / y^3$
8	$x^{-n} = 1/x^n$	$x^{-2} = 1/x^2$
9	$x^{1/n} = \sqrt[n]{x}$	$x^{1/2} = \sqrt[2]{x}$
10	$x^{m/n} = (\sqrt[n]{x})^m$	$x^{2/3} = (\sqrt[3]{x})^2$

Remember

$a^2 + a^2 = 2a^2$ **BUT** $a^2 \times a^2 = a^4$

Simplify $\sqrt{72} = \sqrt{36} \times \sqrt{2}$
$= 6 \times \sqrt{2}$
$= 6\sqrt{2}$

What is the largest square number that is a factor of 72?

Rationalise the denominator

Example:

This is the same as multiplying by 1

$$\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

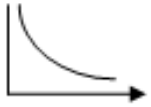
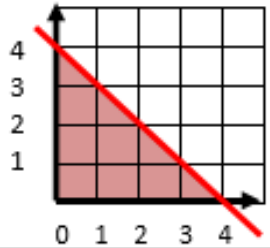
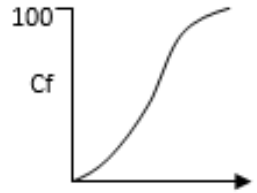
$$= \frac{\sqrt{5}}{\sqrt{(5 \times 5)}} = \frac{\sqrt{5}}{\sqrt{(25)}} = \frac{\sqrt{5}}{5}$$

THE DUSTON SCHOOL

Knowledge Recall

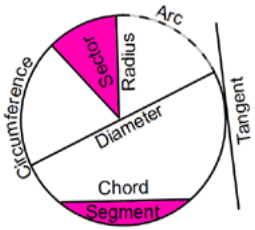
Term1 HW: 1

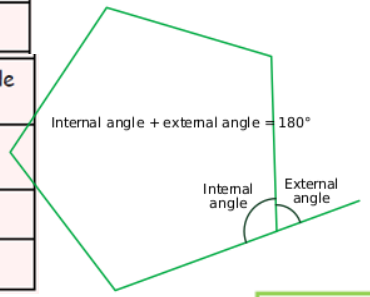
Date Due: _____ **Score to beat:** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{3}{11}$ as a recurring decimal		11. Factorise: $x^2 + 5x + 4$		21. Linear-Quadratic-Cubic-Reciprocal Which function is represented by this graph? 	
2. Write $0.\dot{2}\dot{9}$ as a fraction		12. Factorise: $x^2 - 9$			
3. Work out the balance for £4500 invested for 2 years at 4% per annum		13. Multiply & simplify: $(2a + 1)(a + 3)$		22. What inequality is represented here?  	
4. The value of a car depreciates by 35% per year. Work out the current value of a car bought 2 years ago for £20000.		14. Multiply & simplify: $(3x - 2)^2$			
5. In a '20% off' sale, a coat was £220. Work out the original price.		15. Make r the subject of the formula: $A = \pi r^2$		23. On a spinner: P(3) = ⅜ and the p(4) = ⅛ What is the probability of getting 3 or 4	
6. The cost of a scooter has increased by 20% to £72. Work out the original price.		16. Make t the subject of the formula: $S = at + bt$			
7. Write 84000 in standard form:		17. $S = \frac{u^2 + v^2}{2a}$ Find S when, u = -1, v = -2, a = 2		24. A courgette seed and a pumpkin seed are planted. P(courgette seed germinates) = ⅝ P(pumpkin seed germinates) = ⅜ What is the probability that BOTH seeds germinate?	
8. Write 2.4×10^{-2} as an ordinary number		18. $D = ut + kt^2$ Find D when u = 20, t = 1.2, k = -5			
9. Work out $(4 \times 10^4) \times (2 \times 10^3)$ Give your answer in standard form		19. If $\tan x = \frac{3}{8}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the median reading  	
10. Work out $(6.3 \times 10^7) \div (4.2 \times 10^2)$ Give your answer in standard form		20. Each of these measures is rounded to nearest whole: **a = 5cm and b = 3cm** Calculate the upper bound of a + b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

LINES, ANGLESAND SHAPES KNOWLEDGE ORGANISER

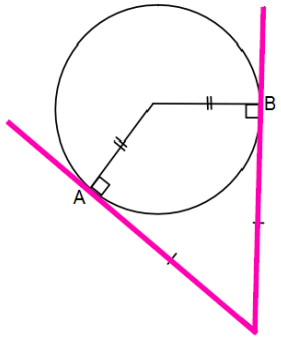
Circumference	The distance around the edge of the circle
Radius	The distance from the centre of the circle to the edge of the circle
Diameter	The distance across the circle from edge to edge, going through the centre
Tangent	A straight line that touches the circle
Chord	A line that touches each edge of the circle but does not go through the centre
Segment	The area created between the circumference and a chord
Arc	Part of the circumference
Sector	A slice of the circle - looks like pizza!





6. Tangents to a Circle

- ✓ A tangent to a circle is always perpendicular to the radius.
- ✓ Two tangents from the same points are equal in length



4	Quadrilateral		90°	360°	2
5	Pentagon		108°	540°	3
6	Hexagon		120°	720°	4
7	Heptagon		128.6°	900°	5

Exterior Angles

The sum of exterior angles in any shape (or polygon) equal 360°

Interior Angles

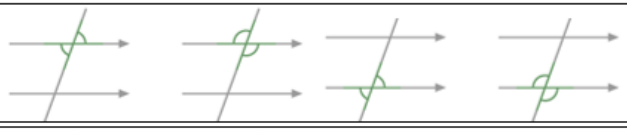
For n sides

Sum of interior angles

= (n – 2) x 180

Each interior angle

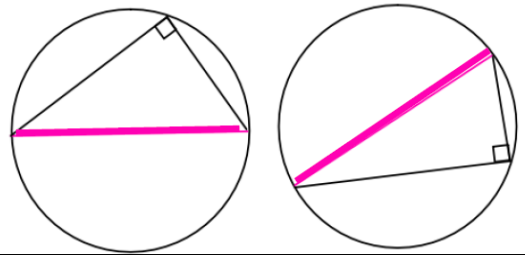
$= \frac{(n-2) \times 180}{N}$

Co-interior/ Supplementary Angles	Make an F shape. Angles are equal.	
Corresponding angles	Make an X shape. Angles are equal.	
Alternate Angles	Make a C-shape. Angles add up to 180°	
Vertically Opposite angles	Make a Z shape. Angles are equal.	

Theorems

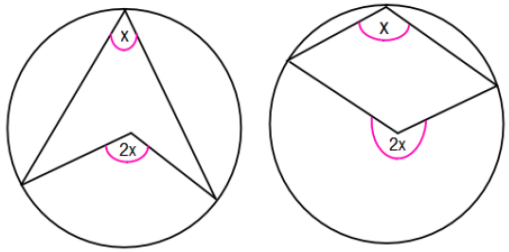
1. Angle in a Semi Circle

- ✓ The angles at the circumference standing on a diameter are equal to 90°



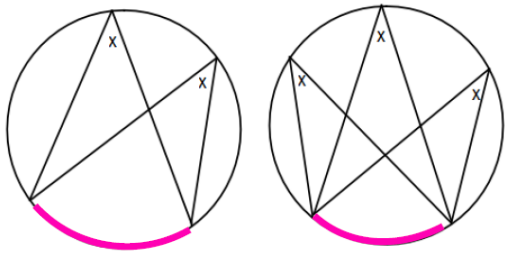
2. Angle at the Centre

- ✓ The angle at the centre is twice the angle on the circumference



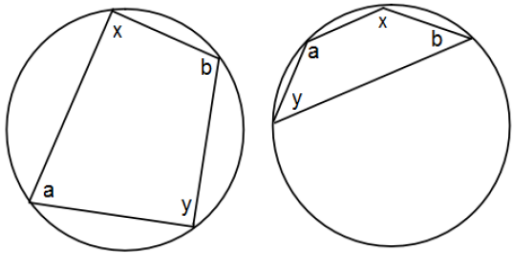
3. Angles on the Same Arc

- ✓ Angles at the circumference standing on the same arc are equal



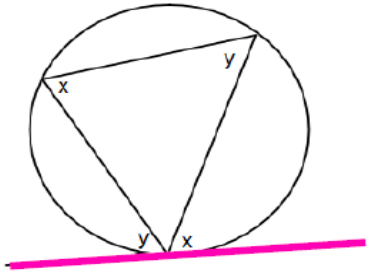
4. Cyclic Quadrilateral

- ✓ Opposite angles in a cyclic quad add up to 180°
- ✓ The vertices MUST be on the circumference



5. Alternate Segment

- ✓ The angle between a tangent and a chord is equal to any angle made by that chord in the alternate segment

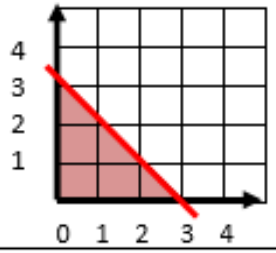
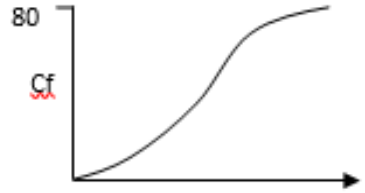


THE DUSTON SCHOOL

Knowledge Recall

Term1 HW: 2

Date Due: _____ **Score to beat:** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{5}{11}$ as a recurring decimal		11. Factorise: $x^2 + 7x + 6$		21. Linear-Quadratic-Cubic-Reciprocal Which function is represented by this graph? 	
2. Write $0.\dot{2}\dot{7}$ as a fraction		12. Factorise: $x^2 - 36$			
3. Work out the balance for £3000 invested for 3 years at 5% per annum		14. Multiply & simplify: $(3x + 4)(x + 1)$		22. What inequality is represented here?  	
4. The value of a car depreciates by 24% per year. Work out the current value of a car bought 3 years ago for £20000.		14. Multiply & simplify: $(2x + 5)^2$			
5. In a '20% off' sale, a coat was £68. Work out the original price.		15. Make r the subject of the formula: $V = \pi r^2 h$		23. On a spinner: P(3) = ⅗ and the p(4) = ⅗ What is the probability of getting 3 or 4	
6. The cost of a scooter has increased by 20% to £144. Work out the original price.		16. Make b the subject of the formula: $S = ab - bc$			
7. Write 63000 in standard form:		17. $S = \frac{u^2 + v^2}{2a}$ Find S when, u = -4, v = -2, a = 5		24. A courgette seed and a pumpkin seed is planted. P(courgette seed germinates) = ⅚ P(pumpkin seed germinates) = ⅓ What is the probability that BOTH seeds germinate?	
8. Write 1.6×10^{-2} as an ordinary number		18. $D = ut + kt^2$ Find D when u = 10, t = 1.4, k = -3			
9. Work out $(3 \times 10^5) \times (2 \times 10^4)$ Give your answer in standard form		19. If $\tan x = \frac{7}{12}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the median reading  	
10. Work out $(6.67 \times 10^8) \div (4.6 \times 10^{-3})$ Give your answer in standard form		20. Each of these measures is rounded to nearest whole: **a = 5cm and b = 3cm** Calculate the lower bound of a + b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

SIMPLIFYING AND SUBSTITUTION KNOWLEDGE ORGANISER

THE DUSTON SCHOOL

Expression: A collection of terms.
(Simplified)

Equation: Expressions that equal one another.
(Solved) (An inequality is a special ≠ equation)

Identity: An equation that is true for all values
(Show) of the variables.

Formula: An identity seen as a general rule.
(Subject)

Function: An expression for which inputs
(Substitute) are used to generate outputs from inputs.

Constants are numbers that stand on their own.

They will not have an "x", "y", "z", or any other variable attached to them. They can be small numbers, like "7", or big numbers, like "23 849".

Variables are symbols that stand for numbers that vary. A variable is usually written as a letter, such as "x", "y", or "z".

The difference between constants and variables is that the value of a variable can change, while the value of a constant stays the same.

SUBSTITUTION

SUBSTITUTION means putting numbers where the letters are.

$x + \frac{x}{2}$

$x = 5$

$5 + \frac{5}{2}$

Key Facts - Adding and Subtracting

Consider the family meal below.

We need to **group the *same items*** together, so when we order through the drive-through its simple.

3b + f + 5c + 3b + s + 2f + m + 2m + mf + c + s + 2b

This would be a **mouthful** to say into the microphone when ordering ... so lets group all the same items together.

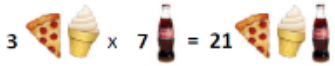
3b + 2b +3b = 8 burgers
s + s = 2 salads
5c + c = 6 cokes
f + 2f = 3 fries
m + 2m = 3 milkshakes
1 McFlurry

If the coefficient is 1 we don't write it.

8b +2s +6c +3f + 3m + mf

Key Facts - Multiplying

5b x 2c = 10bc ✓



3pl x 7c = 21plc ✓

FUNCTIONS

Function $f(x)$ or $x: \rightarrow$ or $y =$ $f(x) = x^2$ function name, input, and the output	A function is a special relationship where each input has a single output. It is often written as "f(x)" where x is the input value.
Composite Function	A function put inside another function e.g. fg(x)
Inverse Function $f^{-1}(x)$	An inverse function goes the other way. e.g. if f(x) = 2x + 3 then $f^{-1}(x) = \frac{x-3}{2}$

EXPANDING

EXPANDING means multiplying all terms together and simplifying.

Single brackets

$3(a + 4) = 3$

	a	+4
3	3a	+12

means:
"3 times (a + 4)"

$= 3a + 12$

Double brackets

$(a + 2)(a + 3) =$

	a	+2
a	a^2	+2a
+3	+3a	+6

means:
"(a + 2) times (a + 3)"

$= a^2 + 5a + 6$

Example of expanding more than two binomials:

$(x + 3)(x - 5)(x + 4)$

First two brackets expand to $x^2 + 3x - 5x - 15 = x^2 - 2x - 15$

$\rightarrow (x^2 - 2x - 15)(x + 4)$

We now must multiply everything in the left hand bracket by x and then by 4

$\rightarrow x^3 - 2x^2 - 15x + 4x^2 - 8x - 60 = x^3 + 2x^2 - 23x - 60$

If you have three binomials times together, you will end up with a *cubic* expression, 4 will make a *quartic* expression and so on.

Multiple binomials - using a grid.

Multiply $(x^2 - 2x - 15)$ by $(x + 4)$

	x^2	$-2x$	-15
x	x^3	$-2x^2$	$-15x$
$+4$	$+4x^2$	$-8x$	-60

$(x + 3)(x - 5)(x + 4) = x^3 - 2x^2 - 23x - 60$

FACTORISING

How to Factorise a Single Bracket

$14y^2 + 21y$

- Find the highest common factor of 14 and 21
- 7 is the highest common factor
- y is the common factor letter
- Factorise... $14y^2 + 7y$ and $21y + 7y$

$7y(2y + 3)$

How to Factorise a Quadratic Equation

A)
How to factorise when the coefficient of x^2 is = 1

$x^2 + 8x + 15$

- Find the **factors** of 15 which sum to 8.
- Factors of 15 are:
- 1, 15, 3, 5
- Only 3 and 5 sum to 8 and multiply to give 15

3 + 5 = 8
3 x 5 = 15

(x + 3)(x + 5)

B)
How to factorise when the coefficient of x^2 is >1

$2x^2 - 5x - 12$

- Multiply 2 x -12 = -24
- Find the **factors** of -24 which **sum** to give you −5 and multiply to give you -24

3 x -8 = -24

3 + -8 = -5

$2x^2 - 8x + 3x - 12$

- **Factorise** the first two terms and the last two terms

$2x^2 - 8x \quad + 3x - 12$

$2x(x - 4) \quad + 3(x - 4)$

- Use the 'common factor term' of **(x - 4)** as the first bracket
- Use what is 'left over' **2x + 3** as the other bracket

(2x + 3) (x - 4)

Difference of Squares

$25v^4 - 16u^4$

$\sqrt{25(v^4)} - \sqrt{16(u^4)}$

$5v^{\frac{4}{2}} - 4u^{\frac{4}{2}}$

$(5v^2 + 4u^2)(5v^2 - 4u^2)$

$4a^4 - 9b^4$

$\sqrt{4(a^4)} - \sqrt{9(b^4)}$

$2a^{\frac{4}{2}} - 3b^{\frac{4}{2}}$

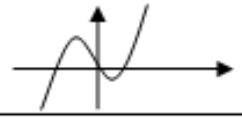
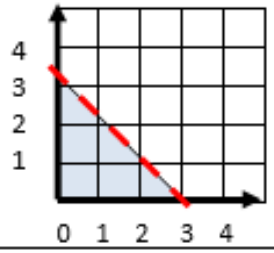
$(2a^2 + 3b^2)(2a^2 - 3b^2)$

THE DUSTON SCHOOL

Knowledge Recall

Term1 HW: 3

Date Due: _____ **Score to beat:** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{5}{6}$ as a recurring decimal		11. Factorise: $x^2 - 6x + 8$		21. Linear-Quadratic-Cubic-Reciprocal Which function is represented by this graph? 	
2. Write $0.\dot{2}\dot{3}$ as a fraction		12. Factorise: $x^2 - 81$			
3. Work out the balance for £5000 invested for 5 years at 7% per annum		14. Multiply & simplify: $(x - 2)(2x + 1)$		22. What inequality is represented here?  	
4. The value of a car depreciates by 18% per year. Work out the current value of a car bought 4 years ago for £20000.		14. Multiply & simplify: $(2x - 7)^2$			
5. In a '20% off' sale, a coat was £120. Work out the original price.		15. Make u the subject of the formula: $v^2 = u^2 + 2as$		23. On a spinner: P(3) = ⅜ and the p(4) = ⅛ What is the probability of getting 3 or 4	
6. The cost of a lamp has increased by 30% to £65. Work out the original price.		16. Make y the subject of the formula: $A = xy + yz$			
7. Write 0.0028 in standard form:		17. $S = \frac{u^2 + v^2}{2a}$ Find S when, u = -3, v = -1, a = 5		24. A courgette seed and a pumpkin seed is planted. P(courgette seed germinates) = ⅛ P(pumpkin seed germinates) = ¾ What is the probability that BOTH seeds germinate?	
8. Write 4.2×10^4 as an ordinary number		18. $D = ut + kt^2$ Find D when u = 20, t = ⅜, k = -5			
9. Work out $(9 \times 10^7) \div (3 \times 10^4)$ Give your answer in standard form		19. If $\sin x = \frac{3}{4}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the lower quartile reading  	
10. Work out $(6.2 \times 10^5) \times (3.8 \times 10^7)$ Give your answer in standard form		20. Each of these measures is rounded to nearest whole: **a = 5cm and b = 3cm** Calculate the upper bound of a - b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	