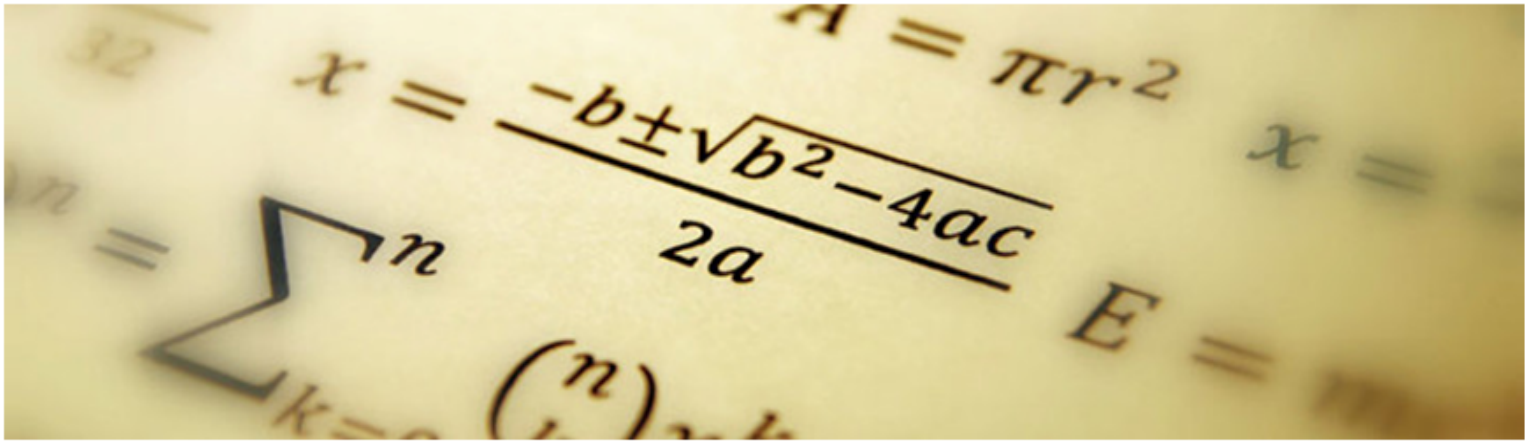
THE DUSTON SCHOOL 4-19

Knowledge Organiser

Maths

Year 10 Term 3

Additional Maths



Contents of Study

Lesson	Topic
1	How can we replace letters with numbers to evaluate an expression?
2	How can we write expressions into their simplest form?
3	What are the laws of indices?
4	How can I expand single brackets?
5	How can we expand single brackets when there is more than one of them in the question?
6	How do I expand a set of two brackets?
7	How can I find a factorise using common factors in algebra?
8	How can I find a factorise using common factors in algebra?
9	How can I factorise a quadratic?
10	How can I solve an equation when I have a variable on one side?
11	How can I solve an equation when I have a variable on both sides?
12	How can I solve an inequality when I have a variable on one side?
13	How can I solve an inequality represent this on a number line?
14	How can I apply my skills of solving to area and perimeter problems?
15	How can I generate a sequence from an nth term?
16	How can I create an nth term from a sequence?

Lessons 1 - 3

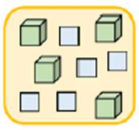
Addition/ Subtraction with indices

Coefficient, Power

$5x^2 + 4x^4$

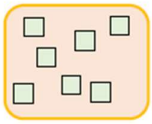
Term, Term

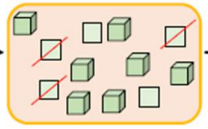
Expression



Each square represents x^2 and each cube represents x^4

Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →  → $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →  → $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \rightarrow 3^7$

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Divide expressions with indices

$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b} \rightarrow \frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$ This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Powers of powers

$$(x^a)^b = x^{ab}$$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

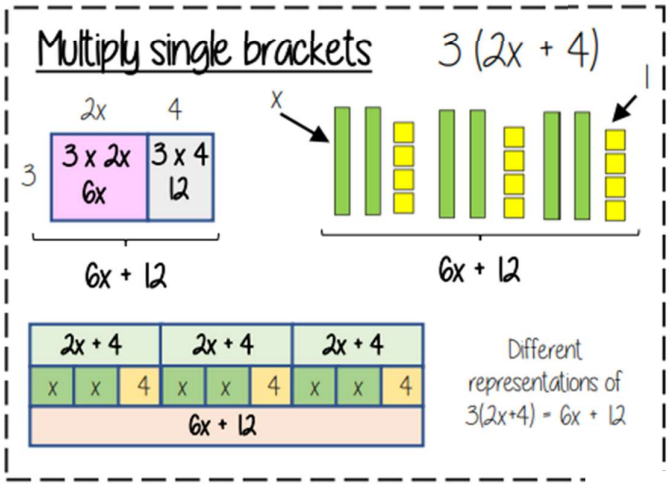
Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Lessons 4 - 6

'Expand' means 'Multiply out'



Example **Expand**

$5(3x + 4)$
$= 5 \times 3x + 5 \times 4$
$= 15x + 20$

x	3x	+4
5	15x	+20

Multiple single Brackets

$2(3x + 1) + 3(x + 5)$
$= 6x + 2 + 3x + 15$
$= 9x + 17$

Expand the bracket(s) first

Example

$-4 \times -2 = (+)8$

$3(2x - 1) - 4(3x - 2)$
$= 6x - 3 - 12x + 8$
$= -4x + 5$

$-3 + 8 = (+)5$

Expanding a double bracket

Method 1 – "smiley face"

Draw loops between each pair and multiply the two values at the end of the loops together

$(2x + 4)(3x + 5)$

$2x \times 3x = 6x^2$

$4 \times 3x = 12x$

$2x \times 5 = 10x$

$12x + 10x = 22x$

$4 \times 5 = 20$

So $6x^2 + 22x + 20$

Method 2 – Separate the brackets

In this method we split the pair of brackets back into single ones

$(2x + 4)(3x + 5)$

$= 2x(3x + 5) + 4(3x + 5)$

$= 6x^2 + 10x + 12x + 20$

$= 6x^2 + 22x + 20$

Method 3 - Grid

Set the expansion out as a multiplication grid

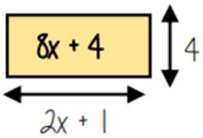
$(2x + 4)(3x + 5)$

	3x	+5
2x	$6x^2$	10x
+4	12x	20

So $6x^2 + 22x + 20$

Lessons 7 - 9

Factorise into a single bracket $8x + 4$



Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Factorise $4a + 20$

$= 4 \times a + 4 \times 5$

$= 4(a + 5)$

The highest common factor of 4 and 20 is 4

Factorise $18x^2 - 27x$

$9x \times 2x - 9x \times 3$

$9x(2x - 3)$

The highest common factor of 18 and 27 is 9. x is the highest common factor of x^2 and x

The highest common factor is 9x

Factorise $12x^2yz - 27xz$

$3xz \times 4xy - 3xz \times 9$

$3xz(4xy - 9)$

The highest common factor of 12 and 27 is 3. xz is the highest common factor of x^2yz and xz

The highest common factor is 3xz

Factorising Double Brackets

Check your signs!

$x^2 + bx + c = (x + \quad)(x + \quad)$

$x^2 + 10x + 16 = (x + 8)(x + 2)$

$x^2 - bx + c = (x - \quad)(x - \quad)$

$x^2 - 10x + 16 = (x - 8)(x - 2)$

$x^2 \pm bx - c = (x + \quad)(x - \quad)$

$x^2 + 6x - 16 = (x + 8)(x - 2)$

$x^2 - 6x - 16 = (x + 2)(x - 8)$

Factorise $x^2 + 9x + 20$

We need 2 numbers that

$x^2 + 9x + 20$

have a sum of (+)9
$4 + 5 = 9$

have a product of (+)20
$4 \times 5 = 20$

$x^2 + 9x + 20 = (x + 4)(x + 5)$

Factorise $x^2 + x - 42$

Sum of (+) 1
$-6 + 7 = 1$

product of - 42
$-6 \times 7 = -42$

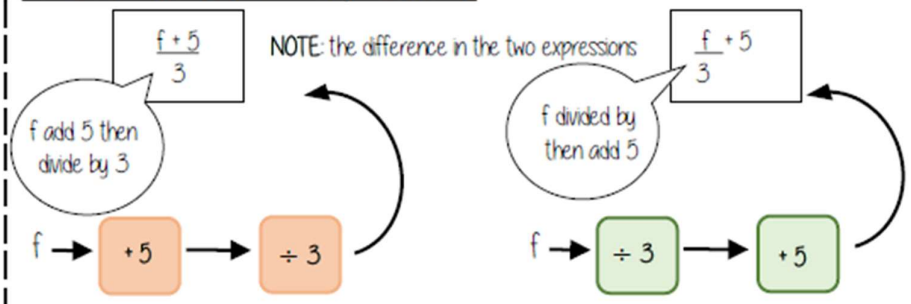
$x^2 + x - 42 = (x - 6)(x + 7)$

Lessons 10 - 11

Using letters to represent numbers

$y + y + y + y$	$20 - h$
$y \times 4$	$\frac{20}{h}$
$4 \times y$	↑
$4y$	20 shared into 'h' number of groups
↑	
4 lots of 'y'	

Find functions from expressions



Sometimes it helps to try to explain the expression in word – and consider what has happened to the input

One-step Equations

1. $y + 14 = 20$
   $-14 \quad -14$
   $y = 6$ ✓

2. $x - 120 = 80$
   $+120 \quad +120$
   $x = 200$ ✓

3. $3n = 12$
   $\frac{3n}{3} = \frac{12}{3}$
   $n = 4$ ✓

4. $\frac{k}{2} = 16$
   $\frac{k}{2} \times 2 = 16 \times 2$
   $k = 32$ ✓

Two-step Equations

$4x + 3 = 35$ (−3, −3)

Subtract 3 from both sides

$4x = 32$ (÷4, ÷4)

Divide both sides by 4

$x = 8$

$5x - 2 = 18$ (+2, +2)

Add 2 to both sides

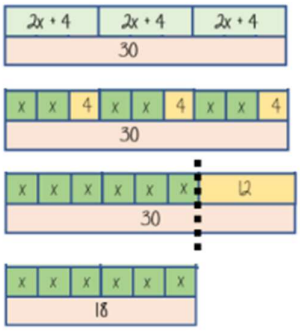
$5x = 20$ (÷5, ÷5)

Divide both sides by 5

$x = 4$

Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$-6 \quad -6$

Substitute to check your answer. This could be negative or a fraction or decimal

x | 3

$x = 3$

Equations with unknown on both sides

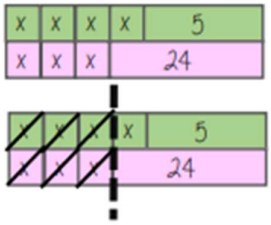
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

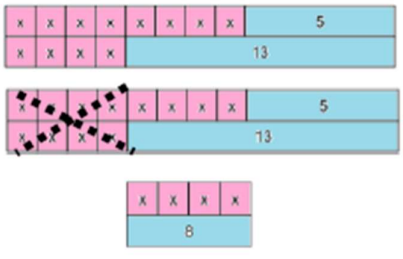
$-5 \quad -5$

$x = 19$



Equations: unknown on both sides R

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$

$-4x \quad -4x$

$4x + 5 = 13$

$-5 \quad -5$

$4x = 8$

$\div 4 \quad \div 4$

$x = 2$

Lessons 12 - 13

Simple Inequalities

< less than

≤ Less than or equal to

> More than

≥ More than or equal to

x < 10
Say this out loud
"x is a value less than 10"

10 > x
Say this out loud
"10 is more than the value"

Note:
x< 10 and 10>x represent the same values

x + 2 ≤ 20
"my value + 2 is less than or equal to 20"
x ≤ 18
The biggest the value can be is 18

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

Form

x → x3 → +2 → 11

3x + 2 > 11

Solve

x ← -3 ← -2 ← 11

x > 3

Check

This would suggest any value bigger than 3 satisfies the statement

3 x 3 + 2 = 11 ✓ 10 x 3 + 2 = 32 ✓

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes ≡

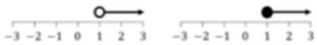
Formula
A rule written with all mathematical symbols
e.g area of a rectangle A = b x h

Solutions on a number line

$x < 1$ $x \leq 1$

Both represent values less than 1

Includes the value 1



$x > 1$ $x \geq 1$

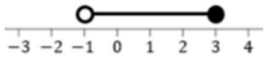
Both represent values more than 1

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



$-1 < x \leq 3$

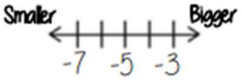
This includes the integer values 0,1,2,3

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$
$+3x \quad +3x$
$2 > 17 + 3x$
$-17 \quad -17$
$-15 > 3x$
$\div 3 \quad \div 3$
$-5 > x$

x is true for any value smaller than -5



✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Inequalities with unknown on both sides

Solving inequalities has the same method as equations

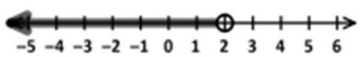
$5(x + 4) < 3(x + 2)$
$5x + 20 < 3x + 6$
$2x + 20 < 6$
$2x < -14$
$x < -7$

Check it!

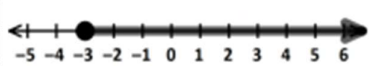
$5(-8 + 4) < 3(-8 + 2)$
$5(-4) < 3(-6)$
$-20 < -18$

✓ -20 IS smaller than -18

$3x - 2 < 4$
$+2 \quad +2$
$3x < 6$
$x < 2$



$2x + 5 \geq -1$
$-5 \quad -5$
$2x \geq -6$
$x \geq -3$

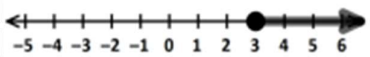


Inequalities: unknown on both sides

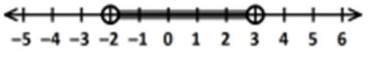
$8x + 5 \leq 4x + 13$ → $x \leq 2$

Any value 2 or less will satisfy this inequality

$5x - 4 \geq 3x + 2$
$-3x \quad -3x$
$2x - 4 \geq 2$
$+4 \quad +4$
$2x \geq 6$
$x \geq 3$



$-1 < 2x + 5 < 9$
$-5 \quad -5 \quad -5$
$-6 < 2x < 4$
$-3 < x < 2$



Lessons 15 - 16

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

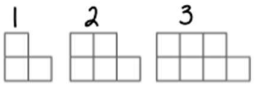
Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Sequence in a table and graphically

Position: the place in the sequence



"The **term** in **position** 3 has 7 squares"

3 5 7

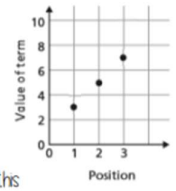
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$ — This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$ — This is not linear as there is a power for n

$2n - 5$ → Substitute the number of the term you are looking for in place of 'n'

eg
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence $3n - 4$?

$3n - 4 = 201$ ← Term to check

Algebraic rule

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$2n^2$ — 2 times whatever n squared is

$(2n)^2$ — 2 times n then square the answer

eg
1st term = 2 x 1^2 = 2
2nd term = 2 x 2^2 = 8
100th term = 2 x 100^2 = 2000

eg
1st term = $(2 \times 1)^2$ = 4
2nd term = $(2 \times 2)^2$ = 16
100th term = $(2 \times 100)^2$ = 40000

$n(n + 5)$ ←

eg
1st term = 1 (1 + 5) = 6
2nd term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

H Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20....

4n

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

$4n + 3$

This is the comparison (difference) between the original and new sequence

Substitution into an expression

$2(x + 3)$

Add 3 to the input then times 2

Put the expression into a function machine

INPUT → +3 → x 2 → OUTPUT

If x = 10
10 + 3 = 13.... 13 x 2 = 26

Forming a sequence

$2(x + 3)$

INPUT	1	2	3
OUTPUT	8	10	12

← The substitution is the 'input' value. The OUTPUT becomes the sequence

Find the nth term: 2, 7, 12, 17

Look at the difference between consecutive terms

7 - 2 = **5** 12 - 7 = **5** 17 - 12 = **5**

So we know the nth term formula will include **5n**

5n	5	10	15	20
	-3	-3	-3	-3
Sequence	2	7	12	17

(5x1 , 5x2, 5x3, 5x4)

The nth term = **5n - 3**

Term 3 Homework 1 Foundation

Use the websites suggested on the overview page to help complete these questions.

Your teacher will let you know when this homework is due. You will go over how to complete these together in class and then you have a go at the next homework which has similar questions.

This will increase your success in these topics in the Summer.

1) Round 0.012784 correct to 2 significant figures

2) Mathswatch clip 106

45 apples are distributed between Adam and Beth in the ratio 2 : 3.
Write the amount each gets in a ratio.

3) Mathswatch clip 741a 71b

$$\frac{3}{4} + \frac{1}{5}$$

4) 0.6×0.8

5) Mathswatch clip 135a

Solve the equation: $4(2x - 2) = 56$

6) Mathswacth clip 159a

Find the gradient and y-axis intercept of the graph

$$y = 3x - 1$$

7) Mathwatch clip 61

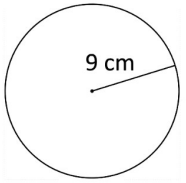
Humanities and Sciences studied by students where each must chose 1 science and 1 humanities option

	Biology	Chemistry	Physics	TOTAL
History	7	17	14	
Geography	2		3	
Ethics		4	3	8
	10	40		70

How many students studied both Chemistry and History?

8) Matchwatch clip 117

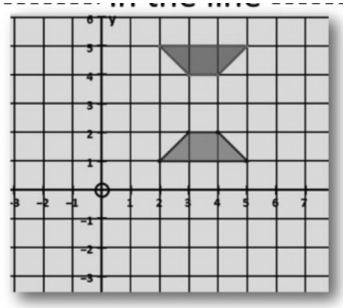
Calculate the area of the circle



Answers correct to 2 decimal places

9) Describe the transformation below

Mathwatch clip 48



10) Mathswatch clip 91

Estimate the answer

$$\frac{48 + 54}{13}$$

11)Matchswatch clip 130a Calculate the mean

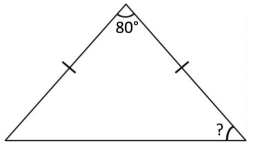
x	frequency
11	2
12	0
13	3
14	3
15	2

12) Factorise fully (Mathswatch clip 94)

$$14y - 21y^3$$

Matshwatch clip 121

13) Calculate the missing angle



14) Work out the angle needed for each type of drink to be able to draw a pie chart
Matchwatch clip 128a

Favourite colour	Frequency
Red	32
Blue	14
Green	18
Yellow	5
Pink	21

Term 3 Homework 2 Foundation

Use the websites suggested on the overview page to help complete these questions.

Your teacher will let you know when this homework is due. You will go over how to complete these together in class and then you have a go at the next homework which has similar questions.

This will increase your success in these topics in the Summer.

1) Round 12.453 correct to 2 significant figures

2) Mathswatch clip 106

32 apples are distributed between Adam and Beth in the ratio 5 : 3.
Write the amount each gets in a ratio.

3) Mathswatch clip 741a 71b

$$\frac{1}{4} + \frac{2}{5}$$

4) 1.3×0.4

5) Mathswatch clip 135a

Solve the equation: $4(3x - 2) = 64$

6) Mathswacth clip 159a

Find the gradient and y-axis intercept of the graph

$$y = 4x - 2$$

7) Mathwatch clip 61

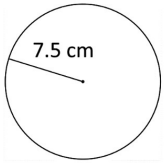
Subjects studied by students

	French	German	Spanish	TOTAL
Male	7	17	8	
Female	2		3	8
TOTAL		20	11	40

How many students studied French?

8) Matchwatch clip 117

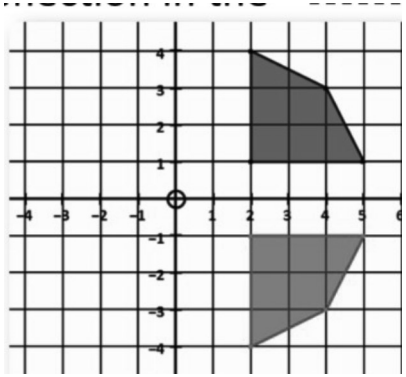
Calculate the area of the circle



Answers correct to 2 decimal places

9) Describe the transformation below

Mathwatch clip 48



10) Mathswatch clip 91

Estimate the answer

$$\frac{487}{12 + 13}$$

11)Matchswatch clip 130a Calculate the mean

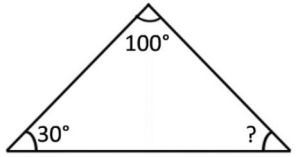
x	frequency
15	4
16	15
17	2
18	0
19	4

12) Factorise fully (Mathswatch clip 94)

$21x^2 + 14x^3$

Matshwatch clip 121

13) Calculate the missing angle



14) Work out the angle needed for each type of drink to be able to draw a pie chart
Matchwatch clip 128a

Favourite Holiday Destination	Frequency
UK	13
France	3
Spain	4
USA	2
Other	8

Term 3 Homework 3 Foundation

Use the websites suggested on the overview page to help complete these questions.

Your teacher will let you know when this homework is due. You will go over how to complete these together in class and then you have a go at the next homework which has similar questions.

This will increase your success in these topics in the Summer.

1) Round 1.02856 correct to 4 significant figure

2) Mathswatch clip 106

28 apples are distributed between Adam and Beth in the ratio 2 : 5.
Write the amount each gets in a ratio.

3) Mathswatch clip 741a 71b

$$\frac{3}{8} + \frac{1}{2}$$

4) 4.5×0.3

5) Mathswatch clip 135a

Solve the equation: $2(2x - 5) = 50$

6) Mathswacth clip 159a

Find the gradient and y-axis intercept of the graph

$$y = 5x - 3$$

7) Mathwatch clip 61

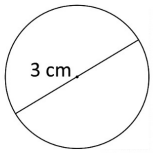
Favourite sport

	Football	Cricket	Tennis	TOTAL
Male	4		18	40
Female	10	26		
TOTAL		44	22	80

How many females chose Cricket?

8) Matchwatch clip 117

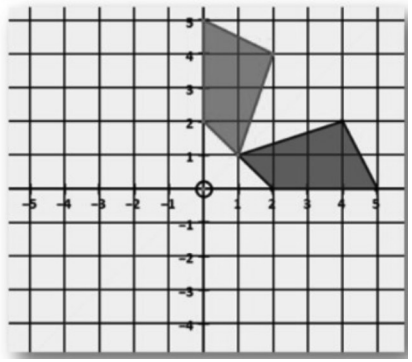
Calculate the area of the circle



Answers correct to 2 decimal places

9) Describe the transformation below

Mathwatch clip 48



10) Mathswatch clip 91

Estimate the answer

$$\frac{734}{23 + 52}$$

11)Matchswatch clip 130a Calculate the mean

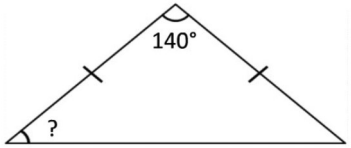
x	frequency
15	4
16	10
17	16
18	12
19	8

12) Factorise fully (Mathswatch clip 94)

$$3x + 15x^3$$

Matshwatch clip 121

13) Calculate the missing angle



14) Work out the angle needed for each type of drink to be able to draw a pie chart
Matchwatch clip 128a

Favourite Type of TV programme	Frequency
Film	11
Soap Opera	14
Music	4
News	7
Documentary	9