Knowledge Organiser
Maths

Year 10 Term 4

Additional Maths
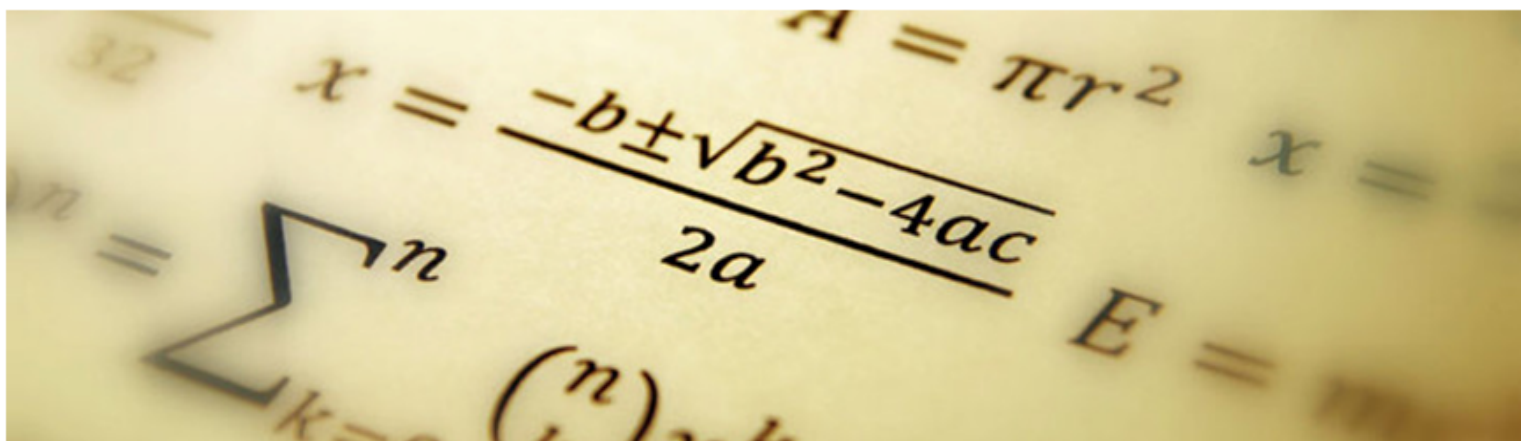
Contents of Study

Lesson	Big Question
1	Ratio notation, simplifying and equivalent ratios
2	Sharing ratios and splitting ratios into amounts
3	Sharing in a ratio – difference of amounts.
4	Ratios as fractions
5	FDP re-cap of amount
6	Worded ratio problems
7	Proportion problems including recipe
8	Proportion problems best buy
9	Worded proportion problems direct proportions
10	Exchange Rates
11	Converting Measure
12	Converting Time, Time tables, bus timetables, worded time problems.
13	Speed Distance Time (Basics)
14	Speed Distance Time (Complex)
15	Density Mass Volume
16	Similarity

Lessons 1 - 4

Representing a ratio

This is the "whole" – boys and girls together

"For every 5 boys there are 3 girls"

$5:3$

This represents the 5 boys   This represents the 3 girls

Double Number Line

This represents the 5 boys

This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs: Cats

$1:2$

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

$6:4$

÷ by 2 ↓

rain   sun

$3:2$

Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiples into them is **2**)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4.

Ratio 1n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1n

$4:20$

$1:5$

The question states that **this part has to be** **1** unit Therefore **Divide by 4**

This side has to be divided by 4 too – to keep in **proportion**

1 the n part does not have to be an integer for this type of question

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James

James: Lucy

$3:4$

£350

Lucy

Find the value of one part

Whole: £350
7 parts to share between (3 James, 4 Lucy)

£350 ÷ 7 = £50

☐ = one part = £50

Put back into the question

James: Lucy

James = 3 × £50 = £150

$3:4$ (× 50 ... × 50)

£150:£200

£350

Lucy = 4 × £50 = £200

Finding a value given 1n (or n:1)

Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue pens

Blue : Red

$5:1$

☐ = one part = 10 pens

Red pens

One unit = 10 pens

Put back into the question

Blue: Red

Blue pens = 5 × 10 = 50 pens

(× 10) $5:1$ (× 10)

$50:10$

Red pens = 1 × 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees

Trees: Flowers

$3:7$

Ratio

Flowers

There are **3** parts for trees

Fraction of trees

$$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} \quad \frac{3}{10}$$

Fraction

Tree parts **3** + Flower parts **7** = **10**

Pi π

Circumference

Diameter

The ratio of a circles circumference to its diameter

Ratios and fractions

For every 3 green counters there are 2 yellow counters

The ratio of green to yellow counters is $3:2$

The fraction of green counters is:

The fraction of yellow counters is:

There are 3 green counters

There are 2 yellow counters

$\frac{3}{5}$

$\frac{2}{5}$

There are 5 counters overall

Combining ratios

B   R   G

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1
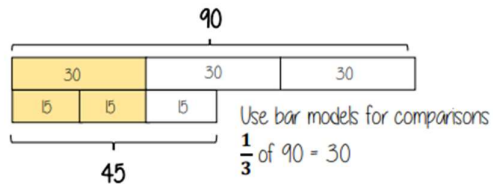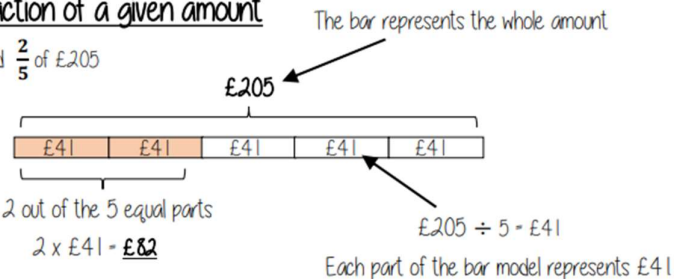
Ratio of Blue to Red to Green

$10:6:3$

Use equivalent ratios to allow comparison of the group that is common to both statements

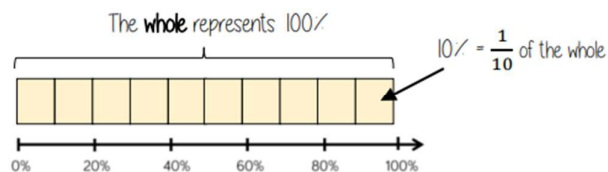Lowest common multiple of the ratio both statements share

Lessons 5 - 6

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

£205

£41	£41	£41	£41	£41

2 out of the 5 equal parts

$2 \times £41 = £82$

£205 ÷ 5 = £41

Each part of the bar model represents £41

90

30	30	30
15	15	15		

45

Use bar models for comparisons

$\frac{1}{3}$ of 90 = 30

$\frac{2}{3}$ of 45 = 30

∴ $\frac{1}{3}$ of $90 = \frac{2}{3}$ of 45

Find the percentage of an amount (Mental methods)

The **whole** represents 100%

$10\% = \frac{1}{10}$ of the whole

0%   20%   40%   60%   80%   100%

$10\% = \frac{1}{10}$ of the whole $50\% = \frac{5}{10} = \frac{1}{2}$ of the whole

$20\% = \frac{2}{10} = \frac{1}{5}$ of the whole $5\% = \frac{1}{20}$ of the whole

Find 65% of 80

80

8	8	8	8	8	8	8	8	8	8
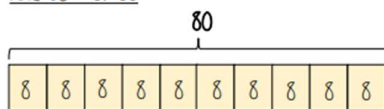
For bigger percentages it is sometimes easier to take away from 100%

Method 1
$65\% = 10\% \times 6 + 5\%$
$= (8 \times 6) + 4$
$= 52$

Method 2
$65\% = 50\% + 10\% + 5\%$
$= 40 + 8 + 4$
$= 52$

Find the percentage of an amount (Calculator methods)

Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$65\% = \frac{65}{100} = 0.65$ ← The multiplier

$0.65 \times 80 = 52$

Using the percent button

Find 65% of 80

Type 65

Press [SHIFT] [( ] (%)

Press [X] 80 and then press =

This brings up the % button on screen
You will see 65%

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

"of" can represent 'x' in calculator methods

| Jack and John share money in the ratio 5 : 3. Jack has £120, how much does John have?	Jack and John share money in the ratio 5 : 3 John has £120, how much does Jack have?
Jack [£24][£24][£24][£24][£24] } £120 John [£24][£24][£24]	Jack [£40][£40][£40][£40][£40] John [£40][£40][£40] } £120
Jack and John share money in the ratio 5 : 3. If they shared a £120 in total, how much did they get each?	**Jack and John share money in the ratio 5 : 3, Jack gets £120 more than John, how much did they get each?**
Jack [£15][£15][£15][£15][£15] } John [£15][£15][£15] } £120	Jack [£60][£60][£60][£60][£60] John [£60][£60][£60] £120
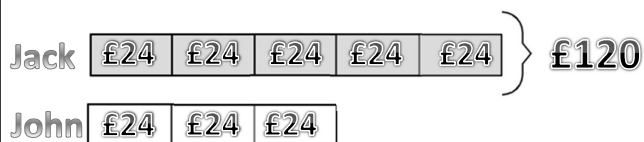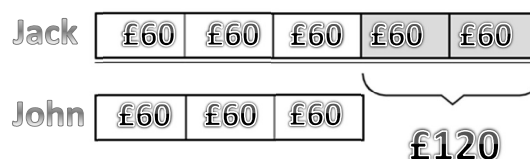
Lessons 7 - 10

Direct Proportion
As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

×0.5 ⟶ 4 cans of pop = £2.40
2 cans of pop = £1.20

×3 ⟶ 4 cans of pop = £2.40
12 cans of pop = £7.20 ⟵ ×3

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs
Compare two variables
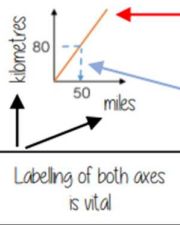
This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Best buys

4 pens costs £2.60

10 pens costs £6.00

• 1 pen costs...   £2.60 ÷ 4 = £0.65   £6.00 ÷ 10 = £0.60

• 1-pound buys...   4 ÷ 2.60 = 1.54 pens   10 ÷ 6 = 1.67 pens

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

Unit Pricing

4 Oranges £1

5 cupcakes £1.20

4 = £1.00 ⟍÷ 2
2 = £0.50 ⟋
1 = £0.25 ⟍÷ 2

5 = £1.20 ⟍÷ 5
1 = £0.20 ⟋

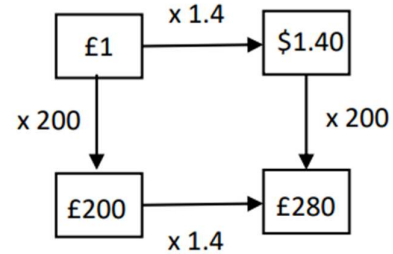**Cost per Unit**

To calculate unit per cost you divide by the cost

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

Best Buys
Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

Shop A
4 cans for £1.20
↓ £1.20 ÷ 4
Cost per item   1 can is £0.30 Or 30p

Shop B
3 cans for 93p
↓ £0.93 ÷ 3
1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

Shop A
4 cans for £1.20
↓ 4 ÷ £1.20
Cost per pound   £1 buys 3.333 cans of pop

3 cans for 93p
↓ 3 ÷ £0.93
£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Exchange Rates

£1 — x 1.4 → $1.40

x 200 ↓   ↓ x 200

£200 — x 1.4 → £280

When making estimates it is also useful to use underlined estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Ratios in 1:n and n:1
This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit
Therefore
Divide by 4

4 : 20
⟶ 1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Lessons 11 - 15

Metric measures

Length Common units of length or distance are

Millimetres (mm) – "Milli" prefix means one thousandth or ÷ 1000
Centimetres (cm) – "Centi" prefix means one hundredth or ÷ 100
Metres (m)
Kilometres (km) – "Kilo" prefix means a thousand × 1000

Average height of a man is 2m
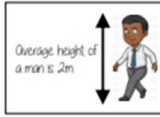
Mass (Weight)

Grams (g)
Kilograms (kg) – "Kilo" prefix means a thousand × 1000
Tonnes (t)

Average weight of an apple is 100g

Capacity

Millilitre (ml) - "Milli" prefix means one thousandth or ÷ 1000
Litre (l)

Average bottle of water holds 500ml

Metric conversions

Length

÷ 10   ÷ 100   ÷ 1000
mm   cm   m   km
× 10   × 100   × 1000

Mass

÷ 1000   ÷ 1000
g   kg   t
× 1000   × 1000
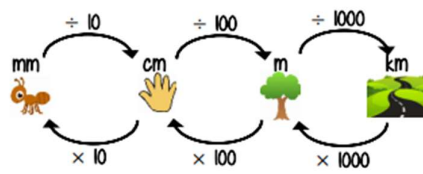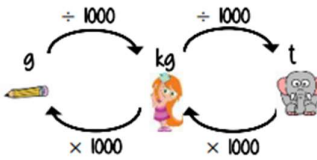
Capacity

÷ 1000
ml   l
× 1000
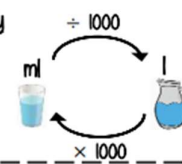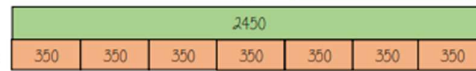
Milli – thousandth
Centi – hundredth
Kilo – thousand

Metric calculations

A package weighs 350g. How much will 7 packages weigh? Give your answers in kilograms

The final weight is in grams

÷ 1000
g   kg

$2450 ÷ 1000 = 245kg$

2450						
350	350	350	350	350	350	350

Calculations tips:
- Do all calculations in the same unit (often the smaller measurement)
- Read for the units of your answer
- Do all conversions of units at the same time
- Represent your image pictorially where possible

Speed, Distance, Time

"per" for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

$$speed = \frac{distance}{time}$$

You can use a double number line to help you calculate distance

Hours: 0   0.5   1   2   2.5
Miles: 0   40   80   160   200

e.g. A boat travels at a constant speed for **2.5** hours. It travels **300** miles.
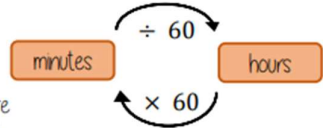
300 miles

◄──── **2.5** hours ────►

Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles

Speed, Distance, Time

Before calculations – make sure you are working in the same units as the speed

÷ 60
minutes   hours
× 60

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$time = \frac{distance}{speed}$$

$$distance = speed × time$$

Density, Mass, Volume

$$density = \frac{mass}{volume}$$

$$volume = \frac{mass}{density}$$

$$mass = volume × density$$
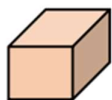
volume of prism = Area of cross section × Depth

Distance – Time graphs

Gradient = speed

The steeper a gradient the faster the speed

$$\frac{10}{5} = 2 \text{ metres per min}$$

Horizontal lines represent staying still

The distance coming closer to home shows the return journey

Units are important
Meters per minute

(graph: Distance from home (metres) vs Time (minutes))

1. There are 26 counters in a bag.

4 of the counters are yellow.
10 of the counters are blue.
8 of the counters are yellow.
The rest of the counters are green.

Maral takes a counter at random from the bag.

Show that the probability that this counter is yellow or green is $^6/_{13}$.

2. Kelly completed a jigsaw in 4 minutes 50 seconds.

Raheem did the same jigsaw in 9 minutes 51 seconds.

Kelly says, "I completed the jigsaw in less than half the time than Raheem did."

Is Kelly right? You must show all your working.

3. A pile of potatoes has a weight of 105 kg.

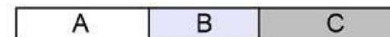Some of the potatoes put into a small bag.

The rest of the potatoes is put into a large bag.

The potatoes in the small bag weighs 35 kg less than the potatoes in the large bag.

What is the weight of the potatoes in the small bag?

4.

Diagram not accurately drawn

A	B	C

Here is a picture of a stick. The length of the stick is in 3 parts, A, B and C.

The total length of the stick is 28 cm.

Part A is 12 cm long and part B is 2 cm long.

Work out the length of part C.

5. Here are three cards. There is a number on each card.

8	9	1

a) Write down the smallest 3-digit number that can be made using each card only once.

b) Write down the largest 3-digit even number that can be made using each card only once.

6. Here is a café menu.

Menu

Cup of tea	75p
Cup of coffee	95p
Fruit Juice	60p
Burger	£2.45
Hot dog	£1.85
Chips	95p

Annasara wants to buy 1 x fruit juice 3 x cup of tea and 3 x burger. she has a £10 note.

Does Annasara have enough money?

You must give reasons for your answer.

1. Two numbers are added together to give 9.

Both of the numbers are factors of 40.

Both numbers are greater than 2.

What are the two numbers?

2. The cost of 1.5 kg of oranges is £0.78.

The total cost of 4.5 kg of oranges and 2.5 kg of melons is £4.79.

Work out the cost of 1 kg of melons.

3. Paulina is going on holiday.

She is going to take out a loan of £800 to help pay for the holiday.

Paulina will have to pay back the £800 plus 20% interest over 12 months.

She will pay back the same amount of money each month.

How much money will Paulina pay back each month?

4. Which of these calculations has the largest answer?

The sum of 13 and 20.

The product of 6 and 5.

The difference between 90 and 73.

Show how you decide.

5. Adeeba is going to buy a new laptop.

The laptop has a price of £360.

Adeeba pays a deposit of 15% of the price of the laptop.

How much money does Adeeba pay as a deposit?

6. Here are 4 numbers.

$\frac{2}{5}$, 0.46, $\frac{9}{20}$, $\frac{9}{25}$,

Write these numbers in ascending order of size.

7.

Item	Price £
Ruler	0.35
Pen	0.14
Rubber	0.17

Ghadi has £20 to spend on pens and rubbers.

She has to buy the same number of pens as rubbers.

What is the greatest number of pens she can buy?

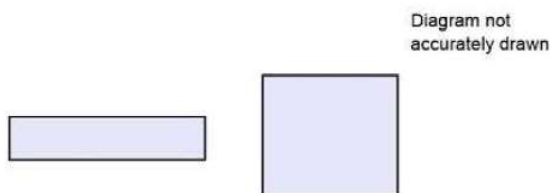**8.** Ayoub and his 4 children are going to London by train.

An adult ticket costs £60 and a child ticket costs £30.

Ayoub has a family railcard which gives $\frac{1}{3}$ off adult tickets and 60% off child tickets.

Work out the total cost of the tickets when Ayoub uses his family railcard.

1. The diagram shows a rectangle and a square.
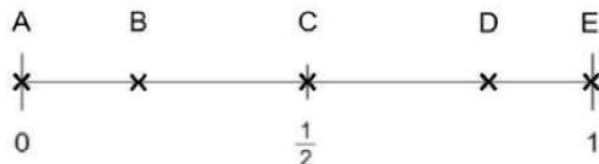
Diagram not accurately drawn

The rectangle is 8 cm long and 2 cm wide.

The perimeter of the rectangle is the same as the perimeter of the square.

Work out the length of one side of the square.

2. Here is a probability scale.

It shows the probability of the events A, B, C, D and E.

```
    A        B        C         D      E
    ✗        ✗        ✗         ✗      ✗
    |_____|_____|_____|_____|
    0                 1/2              1
```

a) Write down the letter of the event that is certain.

b) Write down the letter of the event that is unlikely.

3. Umar and his 2 children are going to Sheffield by train.

An adult ticket costs £42 and a child ticket costs £21.

Umar has a family railcard which gives $\frac{1}{6}$ off adult tickets and 50% off child tickets.

Work out the total cost of the tickets when Umar uses his family railcard.

4. There are 26 counters in a bag.

5 of the counters are green.
9 of the counters are orange.
4 of the counters are purple.
The rest of the counters are pink.

Dawid takes a counter at random from the bag.

Show that the probability that this counter is purple or pink is $\frac{6}{13}$.

5. Which of these calculations has the smallest answer?

The sum of 20 and 16.

The product of 5 and 8.

The difference between 63 and 48.

Show how you decide.

6. There are 27 counters in a bag.

8 of the counters are black.
10 of the counters are white.
7 of the counters are brown.
The rest of the counters are red.

Maria takes a counter at random from the bag.

Show that the probability that this counter is brown or red is $\frac{1}{3}$.

7. Which of these calculations has the largest answer?

The sum of 17 and 11.

The product of 3 and 7.

The difference between 64 and 38.

Show how you decide.