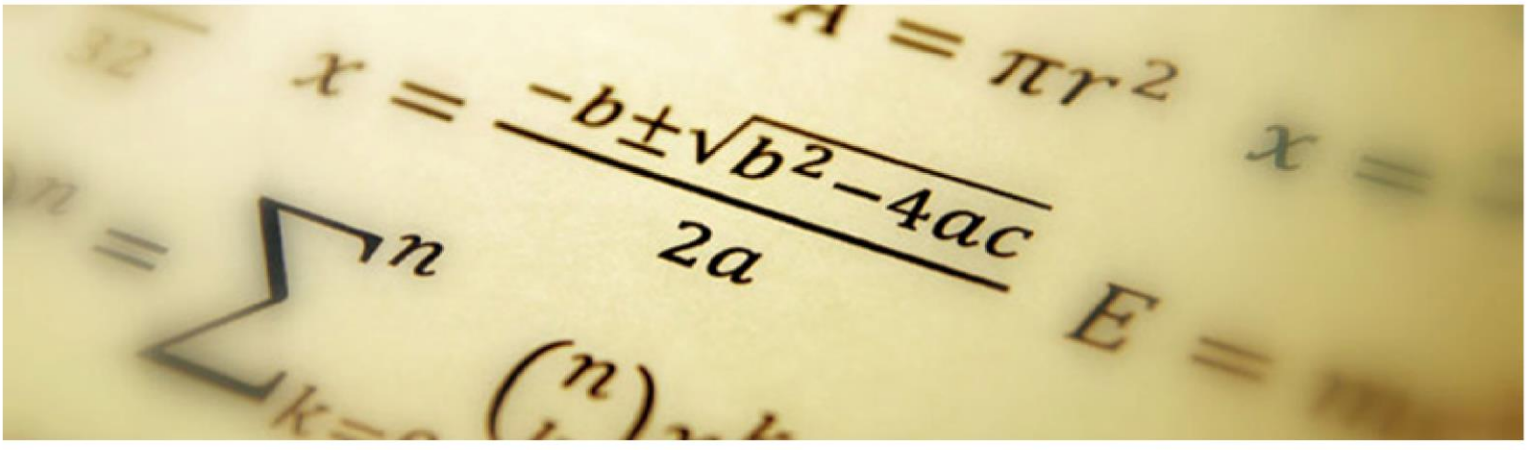
THE DUSTON SCHOOL 4-19

# Knowledge Organiser

## *Maths*

**Year 10 – Term 5**

**Additional Maths**



# Contents of Study

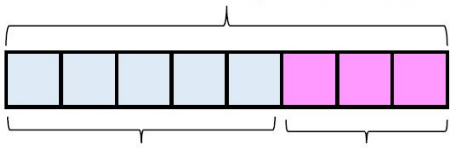
| Lesson | Topic |
|---|---|
| 1 | What is ratio? |
| 2 and 3 | How do I share an amount in a ratio?<br><br>How do I share an amount in a ratio when I only know the difference? |
| 4 | How do I write ratio as fractions? |
| 5 | Can I apply my knowledge ratio to worded problems? |
| 6 | How can I solve an equation when I have a variable on one side? |
| 7 | How can I solve an equation when I have a variable on both sides? |
| 8 | How can I solve an inequality represent this on a number line? |
| 9 | How can I solve an inequality when I have a variable on one side? |
| 10 | How can I generate a sequence from an nth term? |
| 11 | How are proportion questions linked to recipes? |
| 12 | How do I know the best value for money? |
| 13 | What is an exchange rate? |
| 14 | How do I convert my lengths, masses and capacities? |
| 15 | How do I read a timetable? |
| | |
| | |

# Lessons 1 - 5

## Representing a ratio

"For every 5 boys there are 3 girls"

This is the "whole" – boys and girls together

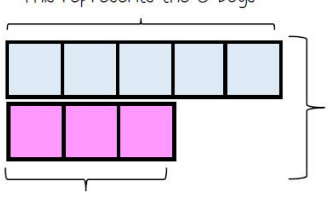


This represents the 5 boys

This represents the 3 girls

5:3

Double Number Line




## Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat. ✗

## Simplifying a ratio

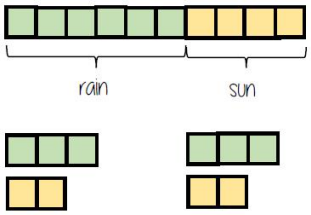
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2




Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiples into them is 2)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

## Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1.

**Show the ratio 4:20 in the ratio of 1:n**

The question states that **this part has to be 1 unit**. Therefore **Divide by 4**

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in **proportion**

*H* the n part does not have to be an integer for this type of question

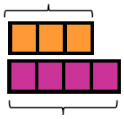
## Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

**Model the Question**

James: Lucy

3 : 4




**Find the value of one part**

Whole: £350

7 parts to share between (3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

**Put back into the question**

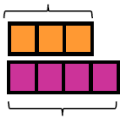
James: Lucy

(x 50) 3 : 4 (x 50)

£150 : £200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200




## Finding a value given 1:n (or n:1)

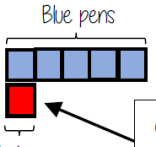
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

**Model the Question**

Blue : Red

5 : 1

□ = one part = 10 pens




One unit = 10 pens

**Put back into the question**

Blue : Red

(x 10) 5 : 1 (x 10)

50 : 10

Blue pens = 5 x 10 = 50 pens

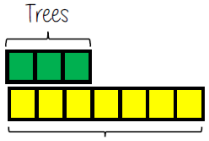
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

## Ratio as a fraction

Trees: Flowers

3 : 7




Ratio

There are 3 parts for trees

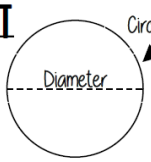
Fraction of trees

Number of parts of in group / Total number of parts = 3 / 10

Fraction

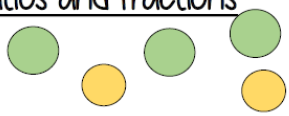
Tree parts 3 + Flower parts 7 = 10

## Pi π




The ratio of a circles circumference to its diameter

## Ratios and fractions



For every 3 green counters there are 2 yellow counters

The ratio of green to yellow counters is 3 : 2

**The fraction of green counters is:**

There are 3 green counters

There are 5 counters overall

3/5

**The fraction of yellow counters is:**

There are 2 yellow counters

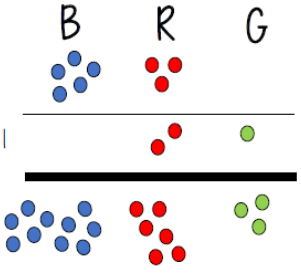
There are 5 counters overall

2/5

## Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1




10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

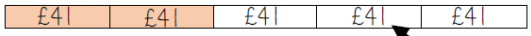
# Lessons 1 – 5

## Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

£205

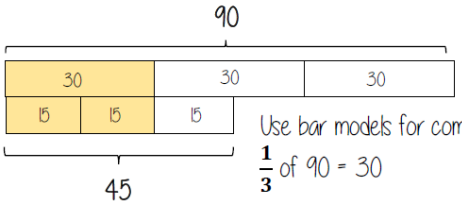



2 out of the 5 equal parts

2 x £41 = £82

£205 ÷ 5 = £41

Each part of the bar model represents £41




Use bar models for comparisons

$\frac{1}{3}$ of 90 = 30

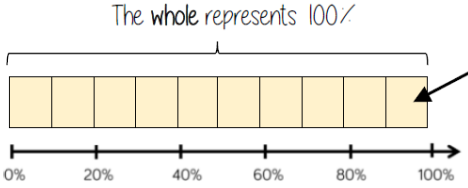
$\frac{2}{3}$ of 45 = 30

$\therefore \frac{1}{3}$ of 90 = $\frac{2}{3}$ of 45

## Find the percentage of an amount (Mental methods)

The whole represents 100%

10% = $\frac{1}{10}$ of the whole




10% = $\frac{1}{10}$ of the whole

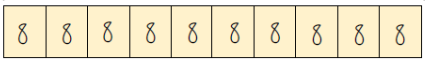
50% = $\frac{5}{10}$ = $\frac{1}{2}$ of the whole

20% = $\frac{2}{10}$ = $\frac{1}{5}$ of the whole

5% = $\frac{1}{20}$ of the whole

Find 65% of 80

80



For bigger percentages it is sometimes easier to take away from 100%

Method 1:

65% = 10% x 6 + 5%

= (8 x 6) + 4

= 52

Method 2

65% = 50% + 10% + 5%

= 40 + 8 + 4

= 52

## Find the percentage of an amount (Calculator methods)

Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

65% = $\frac{65}{100}$ = 0.65 ← The multiplier

0.65 x 80 = 52

Using the percent button

Find 65% of 80

This brings up the % button on screen

You will see 65%

Type 65

Press SHIFT (

Press ×, 80 and then press =

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

"of" can represent 'x' in calculator methods

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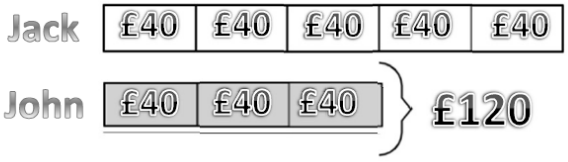
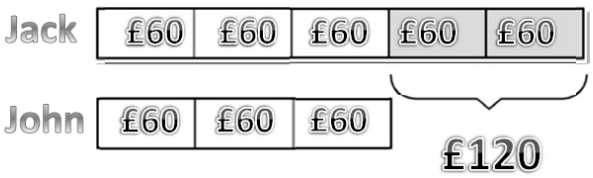
Jack and John share money in the ratio 5 : 3. Jack has £120, how much does John have?


Jack and John share money in the ratio 5 : 3 John has £120, how much does Jack have?




Jack and John share money in the ratio 5 : 3. If they shared a £120 in total, how much did they get each?


Jack and John share money in the ratio 5 : 3, Jack gets £120 more than John, how much did they get each?

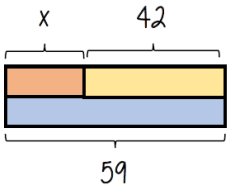


# Lessons 6 – 9

## Solve one step equations (+/-)

There is more to this than just spotting the answer

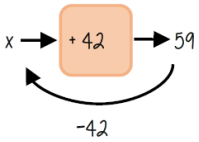
$x + 42 = 59$




$x + 42 = 59$
$42 + x = 59$

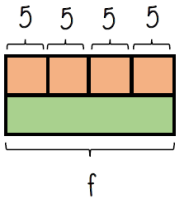
$59 - x = 42$
$59 - 42 = x$

Don't forget you know how to use function machines




## Solve one step equations ( x/÷)

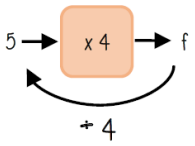
$\frac{f}{4} = 5$




$f \div 4 = 5$
$f \div 5 = 4$

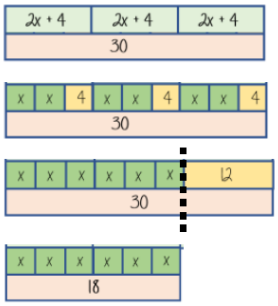
$5 \times 4 = f$
$4 \times 5 = f$

Don't forget you know how to use function machines




## Solve equations with brackets

$3(2x + 4) = 30$




$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \quad -12$

$6x = 18$
$\div 6 \quad \div 6$

$x = 3$

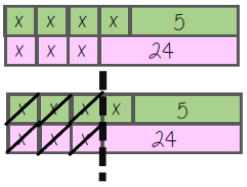
Substitute to check your answer. This could be negative or a fraction or decimal


## Equations with unknown on both sides

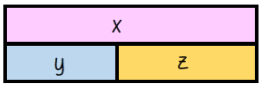
$4x + 5 = 3x + 24$
$-3x \quad -3x$

$x + 5 = 24$
$-5 \quad -5$

$x = 19$




## Rearranging Formulae (one step)




$x = y + z$

Rearrange to make y the subject

$y = x - z$

$y \longrightarrow +z \longrightarrow x$
$y \longleftarrow -z \longleftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging....

Change the subject

Make XXX the subject

Rearrange

$5x - 2 = 18$
$+2 \quad +2$
$5x = 20$
$\div 5 \quad \div 5$
$x = 4$

Add 2 to both sides

Divide both sides by 5

## Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$
$+3 \quad +3$
$4x = 12$
$\div 4 \quad \div 4$
$x = 3$

In a formula (make x the subject)

$xy - s = a$
$+s \quad +s$
$xy = a + s$
$\div y \quad \div y$
$x = \frac{a+s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

Gradient $= \frac{4}{2} = 2$

## Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5 \longrightarrow$

Substitute the number of the term you are looking for in place of 'n'

e.g.
1st term $= 2(1) - 5 = -3$
2nd term $= 2(2) - 5 = -1$
100th term $= 2(100) - 5 = 195$

## Checking for a term in a sequence

Form an equation

Is 201 in the sequence $3n - 4$?

$3n - 4 = 201$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

# Lessons 6 – 9

## Simple Inequalities

< less than

≤ Less than or equal to

> More than

≥ More than or equal to

x < 10
Say this out loud
"x is a value less than 10"

10 > x
Say this out loud
"10 is more than the value"

Note:
x< 10 and 10>x represent the same values

$x + 2 \leq 20$
"my value + 2 is less than or equal to 20"
$x \leq 18$
The biggest the value can be is 18

## Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

**Form**

x → x3 → +2 → 11

$3x + 2 > 11$

**Solve**

x ← -3 ← -2 ← 11

$x > 3$

**Check**

This would suggest any value bigger than 3 satisfies the statement

3 x 3 + 2 = 11 ✓ 10 x 3 + 2 = 32 ✓

## Algebraic constructs

**Expression**
A sentence with a minimum of two numbers and one maths operation

**Equation**
A statement that two things are equal

**Term**
A single number or variable

**Identity**
An equation where both sides have variables that cause the same answer includes ≡

**Formula**
A rule written with all mathematical symbols
e.g. area of a rectangle A = b x h

## Solutions on a number line

$x < 1$ $x \leq 1$

Both represent values less than 1

Includes the value 1

$x > 1$ $x \geq 1$

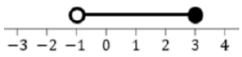
Both represent values more than 1

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1

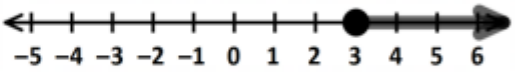


$-1 < x \leq 3$

This includes the integer values 0,1,2,3

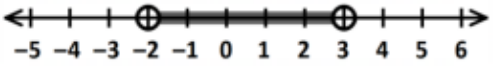
$5x - 4 \geq 3x + 2$
$-3x \quad\quad -3x$

$2x - 4 \geq 2$
$+4 \quad +4$
$2x \geq 6$
$x \geq 3$




$-1 < 2x + 5 < 9$
$-5 \quad\quad -5 \quad -5$

$-6 < 2x < 4$
$-3 < x < 2$




## Inequalities with negatives

**Method 1** Make x positive first

$2 - 3x > 17$
$+3x \quad +3x$
$2 > 17 + 3x$
$-17 \quad -17$
$-15 > 3x$
$\div 3 \quad \div 3$
$-5 > x$

x is true for any value smaller than -5

Smaller ← → Bigger

-7 -5 -3

✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

**Method 2** Keep the negative x

$2 - 3x > 17$
$-2 \quad -2$
$-3x > 15$
$\div -3 \quad \div -3$
$x > -5$

x is true for any value bigger than -5

This cannot be true...

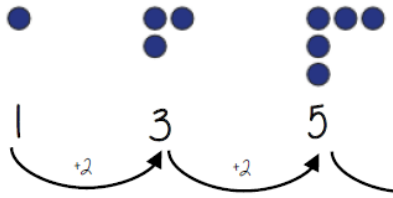
$x < -5$

When you multiply or divide x by a negative you need to reverse the inequality

# Lessons 10

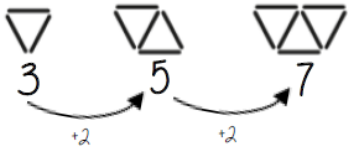
## Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image




What will the next number be? Can you draw this?

## Predict and check terms



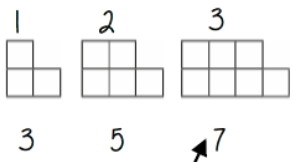

CHECK – draw the next terms


Predictions:
Look at your pattern and consider how it will increase.

e.g. How many lines in pattern 6?
Prediction - 13
If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

## Sequence in a table and graphically

Position: the place in the sequence




"The term in position 3 has 7 squares"

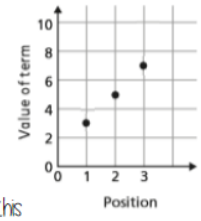
Term: the number or variable
(the number of squares in each image)

In a table

| Position | 1 | 2 | 3 |
|---|---|---|---|
| Term | 3 | 5 | 7 |

+2 +2

Graphically




Because the terms increase by the same addition each time this is linear – as seen in the graph

## Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

## Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?
It increases by adding 4 to each term
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference)
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

## Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?
It increases by multiplying the previous term by 2 – this is a geometric sequence because the constant is multiply by 2
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference)
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

## Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation
Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108...
x3 x3 x3
First term

## H Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20....

4n

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

4n + 3

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

# Lessons 11 – 14

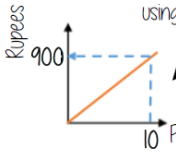
## Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees (×10) → £10 = 900 Rupees (×10)

Currency can be converted using a conversion graph



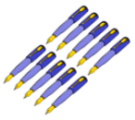

Convert 630 Rupees into Pounds

£1 = 90 Rupees (×7) → £7 = 630 Rupees (×7)

630 ÷ 90 = 7

## Best buys

4 pens costs £2.60



10 pens costs £6.00

| | 4 pens costs £2.60 | 10 pens costs £6.00 |
|---|---|---|
| "1 pen costs..." | £2.60 ÷ 4 = £0.65 | £6.00 ÷ 10 = £0.60 |
| "1-pound buys..." | 4 ÷ 2.60 = 1.54 pens | 10 ÷ 6 = 1.67 pens |

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

## Direct and Inverse Proportion

**Direct Proportion:**

If 5 gallons is the same as 40 pints, calculate how many pints convert to 15 gallons.

5 gallons = 40 pints (×3) → 15 gallons = ? (×3)

**15 gallons = 120 pints**

We could also use the direct proportion graph to convert gallons to pints.

**Inverse Proportion:**

It takes 2 painters 7 days to paint a house. How many days does it take 1 painter to paint the same house?

2 people = 7 days (Divide by 2) → 1 person = ? (Multiply by 2)

**1 person = 14 days**

Multiply on this side as it is the opposite of divide. (We do the opposite for inverse proportion)

## Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

| | Shop A | Shop B |
|---|---|---|
| | 4 cans for £1.20 | 3 cans for 93p |
| | £1.20 ÷ 4 | £0.93 ÷ 3 |
| Cost per item | 1 can is £0.30 Or 30p | 1 can is £0.31 Or 31p |

Shop A is the best value as it is 1p cheaper per can of pop

| | Shop A | |
|---|---|---|
| | 4 cans for £1.20 | 3 cans for 93p |
| | 4 ÷ £1.20 | 3 ÷ £0.93 |
| Cost per pound | £1 buys 3.333 cans of pop | £1 buys 3.23 cans of pop |

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

## Scaling recipes

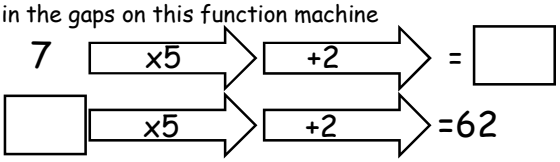
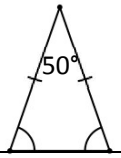
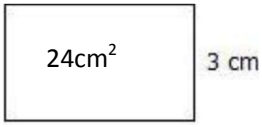
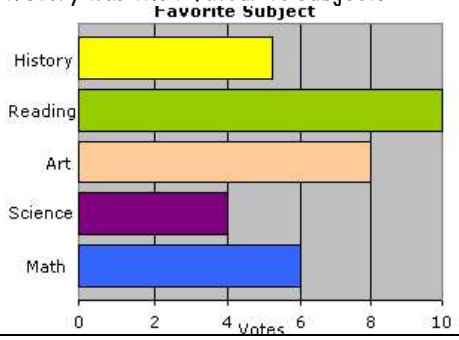
Here is a cake recipe for 6 people.

3 eggs
300g flour
150g sugar

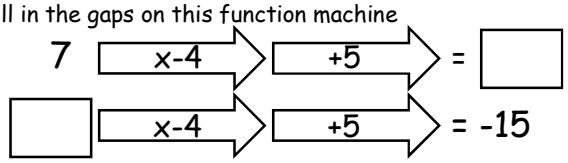
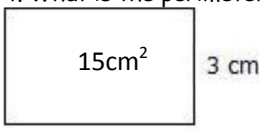
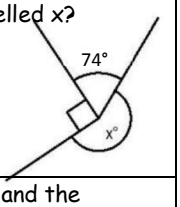
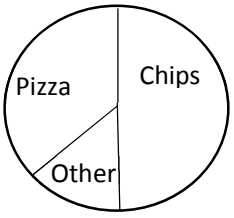
What would you need for 8 people?

| | 6 | ÷3 → 2 | ×4 → 8 |
|---|---|---|---|
| eggs | 3 | 1 | 4 |
| flour | 300g | 100g | 400g |
| sugar | 150g | 50g | 200g |

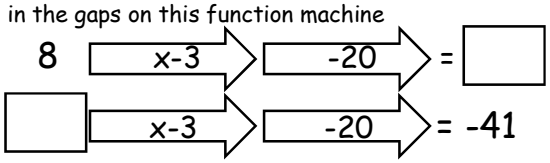
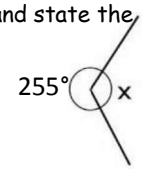
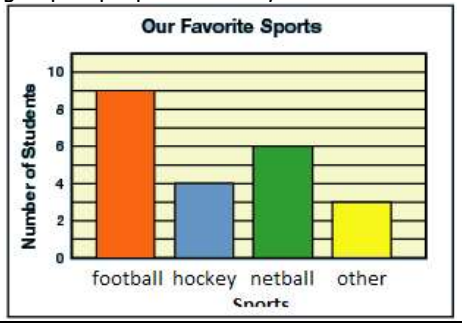
Mixed Topic Homework Sheet 13

| | |
|---|---|
| 1. Calculate 20 ÷ (3 + 7) = | 2. Fill in the gaps on this function machine<br>  |
| 3. Write the following ratio in its simplest form,<br>12:60 | 4. If the perimeter of a shape is 36cm, what could the width and length be? |
| 5. If I have 35 pencils and 7 are blue, what is the probability of me choosing a blue pencil? | 6. What is the median of the following set of numbers?<br>17, 19, 26, 29, 32, 33, 40, 43, 45, 58, 61 |
| 7. List all the factors of the number 60 | 8. Write the next three terms of the following sequence:<br>15, 18, 21, ____, _____, _____ |
| 9. If I need 8 bags of cement to make 40kg of concrete, how many bags would I need to make 100kg of concrete. | 10. Calculate the size of the missing angles, and state the reasons for your answer.<br>  |
| 11. Find the mean for the following set of numbers:<br>15, 15, 16, 19, 21, 25, 28, 31, 34 | 12. If the probability picking a green marble from a bag of green and red marbles is 0.45, what is the probability of choosing a red marble? |
| 13. Write the following decimals from smallest to largest:<br>0.203 0.14 0.014 0.9 0.09 | 14. Simplify 4a x 6 + 5b x 3 |
| 15. What is the width of the shape below?<br>  | 16. If the probability of a pin landing point up is 0.4, how many times would I expect it to land point up if I were to drop the pin 200 times? |
| 17. What is 55% of 340 | 18. Solve the following equation: 4g - 5 = 27 |
| 19. How many litres is 53830 ml? | 20. Based on the bar chart below, how many people said history was their favourite subject?<br>  |
| Total: /20 | Personal Target: |

Mixed Topic Homework Sheet 14

| | |
|---|---|
| 1. Calculate (4 + 5) x (4 – 7) | 2. Fill in the gaps on this function machine<br>  |
| 3. Write the following ratios in their simplest forms,<br>5:35 3:39 15:75 | 4. What is the perimeter of the following shape?<br>  |
| 5. If I have 8 apples, 8 pears and 2 oranges, what it the probability of choosing a pear? | 6. What is the mode of the following set of numbers?<br>4, 5, 8, 5, 2, 6, 4, 5, 4, 8, 6, 4 |
| 7. List 3 multiples of 7. | 8. What is the nth term rule of the following sequence?<br>3, 5, 7, 9 |
| 9. If a company need 50 potatoes to make 125 packets of crisps, how many potatoes would they need to make 250 packets of crisps? | 10. What is the size of the missing angle labelled x?<br>  |
| 11. Find the median for the following set of numbers:<br>25, 22, 13, 36, 17, 25, 34, 22, 19 | 12. If the probability winning a game is 0.15, and the probability of drawing is 0.4, what is the probability of losing? |
| 13. Write the following in order from smallest to largest:<br>0.13, 1/8, 2/5, 0.35, 31% | 14. If a = 4 and b = 7, what would 3a + 4b be equivalent to? |
| 15. What is the area of a square with a side length of 12cm. | 16. If the probability of the next car I see being red is 0.3, how many of the next 300 cars I see would I expect to be red? |
| 17. Increase 90 by 20% | 18. Solve the following equation:<br>5a – 12 = 28 |
| 19. How many centimetres is 67.5 metres? | 20. If the pie chart below represents favourite dinners of 300, how many people chose chips?<br>  |
| Total: /20 | Personal Target: |

Mixed Topic Homework Sheet 15

| | |
|---|---|
| 1. Calculate 16 – 3 + (5-2) = | 2. Fill in the gaps on this function machine<br>  |
| 3. If the ratio of red socks to blue socks is 3:8, how many red socks would I have if I had 24 blue socks? | 4. What is the perimeter of a football pitch of length 104m and width of 65m? |
| 5. If I have 55 counters, 20 are red, 5 are green and the rest are blue, what is the probability of not choosing a red counter? | 6. What is the mode of the following set of numbers?<br>64, 58, 64, 72, 55, 64, 55, 72, 65, 55 |
| 7. Express 48 as a product of it's prime factors. | 8. Write the nth term rule of the following sequence.<br>-6, 5, 16, 27, 38 |
| 9. If I need 65g of sugar to make 5 donuts, how much sugar do I need to make 20 donuts? | 10. Calculate the size of the angle labelled x, and state the reason for your answer.<br>  |
| 11. Find the median for the following set of numbers:<br>2, 8, 9, 2, 4, 5, 7, 6 | 12. If the probability of choosing a strawberry from a bag of strawberries, pears and apples is 0.42, and the probability of choosing a pear is 0.17, what is the probability of choosing an apple? |
| 13. Write the following from smallest to largest:<br>$\frac{3}{4}$ 0.6 2/3 0.7 4/5, 63% | 14. If x = 7 and y = -2, what is the value of<br>5x + 2y + 2x + 5y |
| 15. What is the width of a square with an area of 64cm2 | 16. What is the probability of choosing a black card from a standard pack of cards? |
| 17. Decrease 160 by 15% | 18. Solve the following equation: 6(x + 3) = 72 |
| 19. How many metres are in 9.86km? | 20. If the bar chart below represents favourite sports of a group of people, how many did not choose hockey?<br>  |
| Total: /20 | Personal Target: |