THE DUSTON SCHOOL
Knowledge itself is power

Year 10 Maths Intermediate Knowledge Booklet Term 4

Name:

Class:

Order	Unit	Links	Pre-requisite skills
1	Integers, powers & roots		
2	Lines, angles & shape		
3	Simplifying & substituting	Unit 1	Using powers, listing factors, understanding product / sum.
4	Area and perimeter	Unit 2	Forming expressions for area/perimeter algebraically through use of brackets, correct notation and simplifying expressions.
5	Calculations & Accuracy	Unit 1	Understanding numbers.
6	Construction and LOCI	Unit 2	Measuring angles for bearings, parallel line angle facts.
7	FDP	Unit 1	Using powers, understanding lowest common multiples.
8	Sequences, functions and graphs	Unit 3/5	Substituting into a function applying BIDMAS to calculate coordinates, factorising for roots of quadratics, understanding powers and all 4 operations with negatives.
9	Ratio & Proportion	Unit 1/7	Decimals/powers as multipliers, calculating/understanding fractions as parts.
10	Transformations	Unit 2/8	Identifying 90/180/270 degrees, plotting mirror lines of basic functions.
11	Pythagoras and Trigonometry	Unit 1/2/3/4/5	Powers/surds, types of triangles, use in area/perimeter problems to find required lengths, rounding answers.
12	Forming and solving	Unit 3/4	Properties of 2d shapes, angle facts including polygons & parallel lines, algebraic notation and simplifying, forming expressions.
13	Measures	Unit 1/7	Calculating, multiplying decimals and powers of 10 for metric conversions.
14	Volume and Surface area	Unit 4/5/13	Area of 2d shapes, rounding/calculating with bounds, conversion of units (length/area/volume), calculating missing sides using pythagoras/ trigonometry.
15	Probability	Unit 1/7	Types of numbers, calculating with fractions & decimals.
16	Inequalities	Unit 12/8/5/7	Solving equations, rounding, plotting graphs for regions, calculating with fractions.
17	Statistics	Unit 1/6/9/16	Using a protractor for pie charts, proportion to calculate angles for a pie chart, use of inequality symbols for recording data.

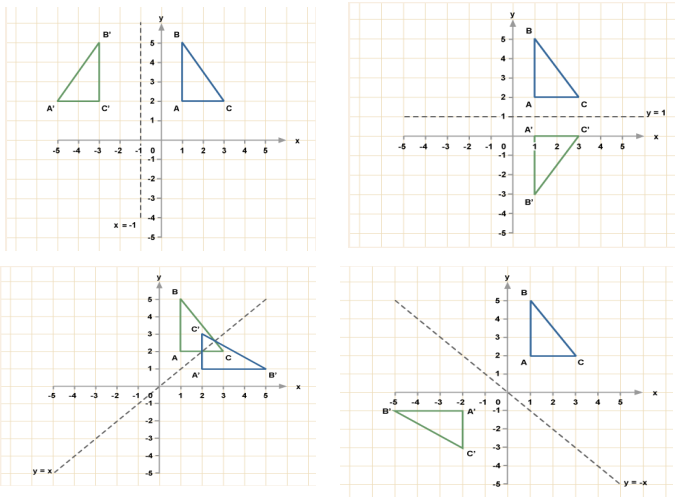
Homework 1 Due	
Homework 2 Due	
Homework 3 Due	

Year 10 - Term 4: Intermediate

Overview	**Learning Objective**		
Topic: Transformations **Big Questions** - True/Never/Sometimes: - Reflected shapes are the same size and shape as the original shape. - Rotated shapes are the same size and shape as the original shape. - Translated shapes are the same size and shape as the original shape.	- Enlarge a shape by a positive integer scale factor from a given centre. - Enlarge a shape by a positive fractional scale factor given a centre.	- Describe all four transformations. - Combined transformations. (Rotations which is the same as an enlargement.) - Introduction to vectors. (Add, subtract and multiply vectors)	- Enlarge a shape by a negative scale factor given a centre - Describe the changes and invariance achieved by combinations of rotations, reflections and transformations.
Topic: Pythagoras and Trigonometry **Big Questions** - What is the same/ different about three triangles with sides 3, 4, 5 and 6, 8, 10 and 5, 12, 13 - True/Never/Sometimes: - You can use trigonometry to find the missing length/ angle in triangles - True/Never/Sometimes: - Pythagoras's Theorem can be used to find the lengths of sides in triangles	- Use Pythagoras' Theorem to calculate the length of the hypotenuse of a right-angled triangle. - Use Pythagoras' Theorem to calculate the length of any side of a right-angled triangle. - Use Pythagoras' Theorem to calculate the height of an isosceles triangle. - Use Pythagoras' Theorem in practical problems.	- Find the distance between two coordinates. - Know the exact values of sine, cosine and tangent at key angles (0, 30, 45, 60, 90 degrees). - SOHCAHTOA to calculate missing sides in right-angled triangles. - SOHCAHTOA to calculate missing angles in right-angled triangles. - Use SOHCAHTOA in practical problems.	
Topic: Forming and solving equations **Big Questions** - Show me an example of a formula that has the value 7 when a = 2 and b = 3 - What is the same/ difference between: 5P + 3Q = 12 and 10P + 7Q = 24 - How can you change v = u + at so that (i) u is the subject (ii) t is the subject?	- Solve linear equations -Solve all forms of linear equations with unknowns on both sides. - Derive more complex formulae & equations from words. (including shape) -Mathematical reasoning. (is the sum of two odd number odd?)	- Factorise and **solve** quadratics in the form $ax^2 + bx + c = 0$ where a = 1. - Solve linear simultaneous equations.	- Factorise and solve quadratics in the form $ax^2 + bx + c = 0$ where a > 1. -Rearrange formulae where the variable appears twice.

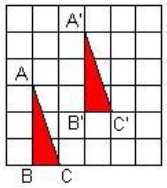
TRANSFORMATIONS KNOWLEDGE ORGANISER

REFLECTION



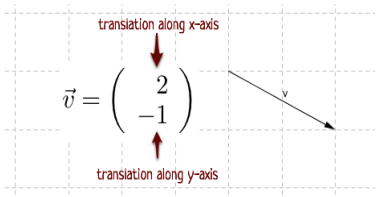
Name	Shape	Order of Rotational Symmetry
Parallelogram		2
Regular Polygon with n sides	Examples:	n
Rhombus		2
Circle		Unlimited
Trapezium		None
Kite		None

TRANSLATION



Each point moved 2 to the right and 2 up.

The vector describing this translation is: $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$



Equal vectors

If two vectors have the **same magnitude** and **direction**, then they are **equal**.

Adding vectors

Look at the graph below to see the movements between **PQ**, **QR** and **PR**.

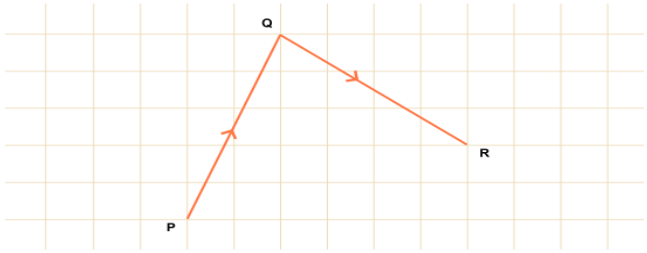
$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

Vector $\overrightarrow{PQ}$ followed by vector $\overrightarrow{QR}$ represents a movement from **P** to **R**.

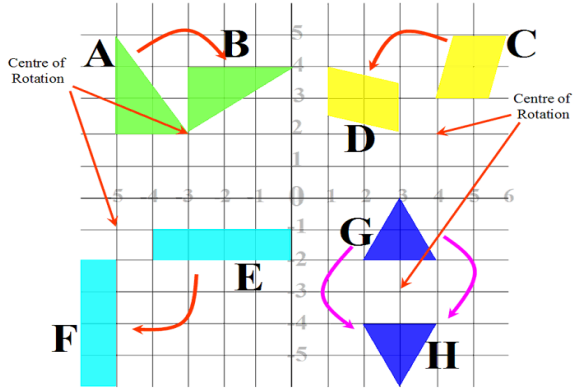
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

Written out the vector addition looks like this

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$



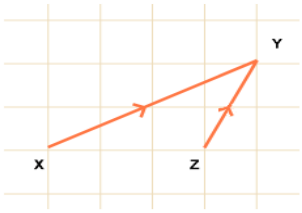
ROTATION



Subtracting vectors

Subtracting a vector is the same as adding a negative version of the vector (remember that making a vector negative means reversing its direction).

$\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$



Look at the diagram and imagine going from X to Z. How would you write the path in vectors using only the vectors $\overrightarrow{XY}$ and $\overrightarrow{ZY}$?

You could say it is vector $\overrightarrow{XY}$ followed by a backwards movement along $\overrightarrow{ZY}$.

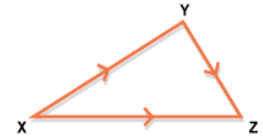
So we can write the path from X to Z as

$\overrightarrow{XY} - \overrightarrow{ZY} = \overrightarrow{XZ}$

Written out in numbers it looks like this:

$\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

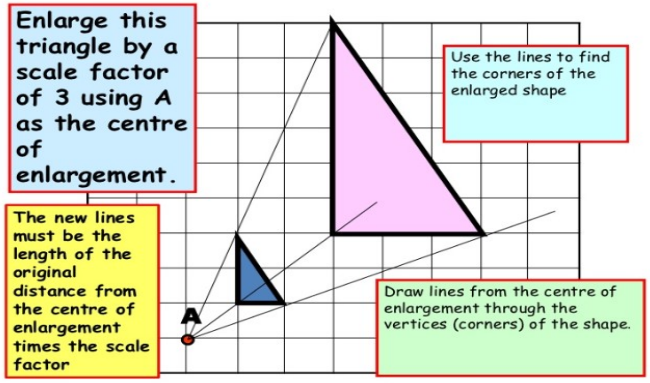
Resultant vectors

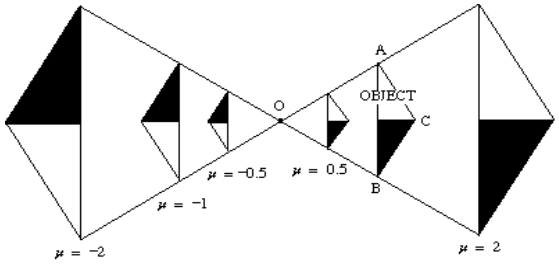


To travel from **X** to **Z**, it is possible to move along vector $\overrightarrow{XY}$ followed by $\overrightarrow{YZ}$. It is also possible to go directly along $\overrightarrow{XZ}$.

$\overrightarrow{XZ}$ is therefore known as the **resultant** of $\overrightarrow{XY}$ and $\overrightarrow{YZ}$.

ENLARGEMENT





Fractional scale factors

If we 'enlarge' a shape by a scale factor that is between -1 and 1, the image will be **smaller** than the object

Negative scale factors

An enlargement using a negative scale factor is similar to an enlargement using a positive scale factor, but this time the image is on the other side of the centre of enlargement, and it is upside down.

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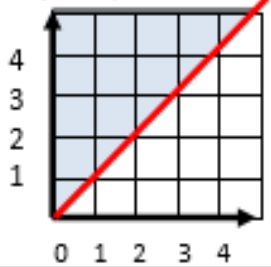
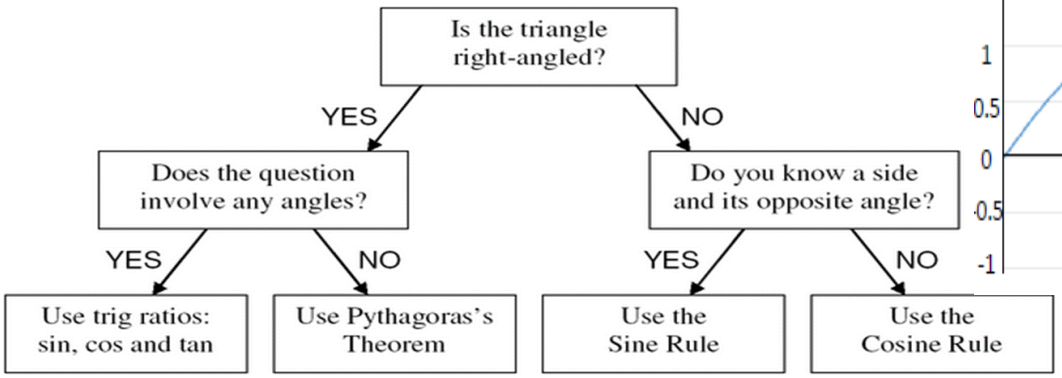
Knowledge Recall **Term 4 HW: 1**

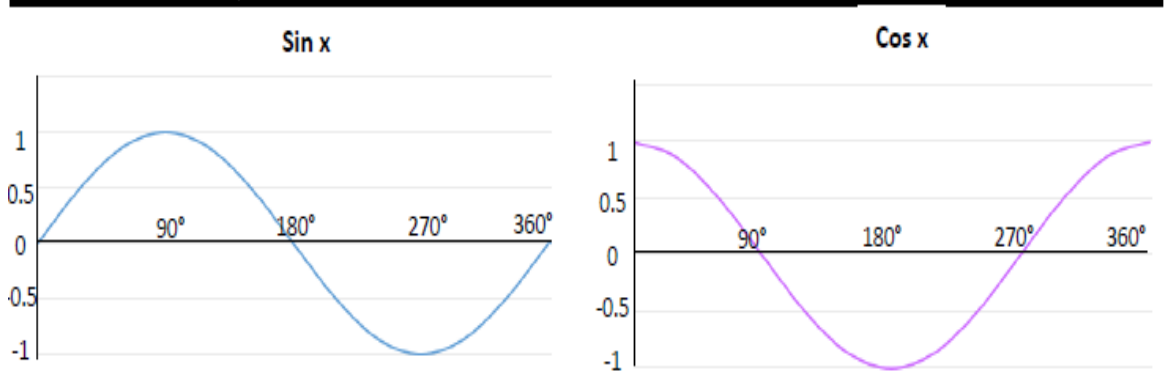
Date Due _____ **Score to beat** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. To increase an amount by 24%, what single multiplier would you use?		11. Expand & simplify: $2(2x + 3) + 2(x - 2)$		21.Volume is $144cm^3$, Find x? 	
2. Divide 72 in the ratio of 7 : 2		12. Solve: $x + 8 \geq 5$			
3. Work out: $1\frac{2}{3} \div \frac{3}{4}$		13. Make a the subject of the formula: **T= a - 2**		22. 5.7 is rounded to one decimals place. Write down the maximum possible it could have been.	
4. Estimate the answer to: 7.9 x 0.67		14. Write down the nth term of this sequence: -1 3 7 11 15 ...			
5. Work out the LCM of 6 and 9		15. If $y = x^2 + 2x$, find the value of y when x = -1		23. The mass of a bar of chocolate is 1800g. The density of the chocolate is $9g/cm^3$ What is its volume?	
6. Write $0.\dot{3}\dot{6}$ as a fraction		16. Factorise: $y^2 - 169$			
7. Work out the balance for £720 invested for 4 years at 5% per annum		17. Multiply & simplify: $(3x - 1)(3x + 1)$		24.What inequality is represented here? 	
8. The cost of a fridge has increased by 15% to £828. Work out the original price.		18. Make s the subject of the formula: $v^2 = u^2 + 2as$			
9. Write 41500 in standard form:		***Give your answer correct to 3sf*** 19. $A = \pi r^2 - \pi rs$ Find A when, r = 6.5, s =2.5		25. On a spinner: P(3) = ⅛ and the p(4) = ⅜ What is the probability of getting 3 or 4	
10. Work out $(7x10^3) \times (8 \times 10^2)$ Give your answer in standard form		*B.14* 20. If $\tan 63^0 = \frac{x}{4}$, find x (3sf)			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

Pythagoras' and Trigonometry



BASIC Sine and Cosine Graphs



Cosine Rule

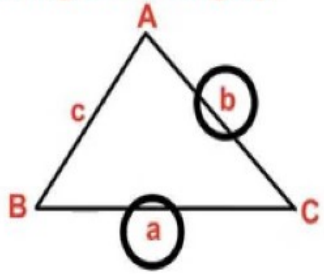
The Cosine Rule can be used in any triangle

$a^2 = b^2 + c^2 - 2bc \cos A$ Used to calculate an unknown **SIDE**

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ Used to calculate an unknown **ANGLE**

Area of a Triangle (Non Right-Angled Triangle)

$$\text{Area} = \frac{1}{2} a b \sin C$$



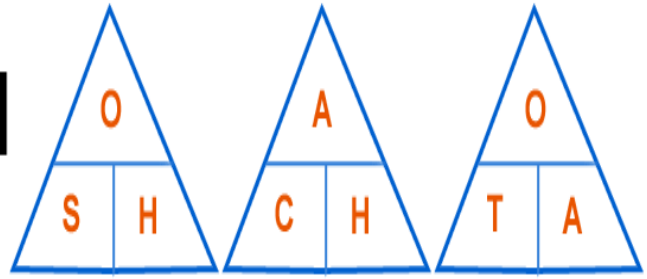
This formula finds the area of a non right-angled triangle from 2 sides

Key Facts - Exact Trigonometric Values

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$

Key Facts – Pythagoras' Theorem

$a^2 + b^2 = c^2$	Where C is the hypotenuse
$c^2 - a^2 = b^2$	Where the unknown side is the shorter side (aka 'leg')



Sine Rule

The Sine Rule can be used in any triangle

Used to calculate an unknown **SIDE**

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

Used to calculate an unknown **ANGLE**

$$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

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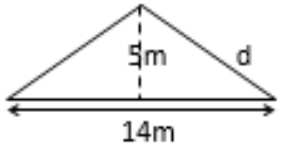
Knowledge Recall **Term 4 HW: 2**

Date Due _____ **Score to beat** _____

Section A:Number		Section B: Algebra		Section C: Using and applying	
1. To increase an amount by 3.2%, what single multiplier would you use?		11. Expand & simplify: $x(x + 2) + x(x + 3)$		21.  Find 'd' to 1DP:	
2. Decrease £750 by 18%		12. Factorise: $6m - 14$			
3. Divide 360 in the ratio of 5 : 7		13. Simplify: $2g^3 \times 3g^2$		22. 40 is rounded to the nearest whole. Write down the maximum possible length it could have been.	
4. Galina and Hiran shared 36 sweets. Galina had 12 more sweets than Hiran. What was the ratio of sweets shared in its simplest form.		14. Solve: $4x \leq 10$			
5. Work out: $1\frac{4}{5} - \frac{3}{4}$		15. Make d the subject of the formula: $A = cd$		23. A block of copper weighs 2160g and has a volume of $240cm^3$. What is the density of the copper?	
6. Work out: $2\frac{2}{5} \div \frac{3}{4}$		16. Work out the value of: $xy + 5$ When $x = 2$ and $y = 3$			
7. Round off 0.482 to one significant figure		17. Write down the nth term of this sequence: 1 7 13 19 25 ...		24. In an experiment the colours of 50 cars passing was recorded. 17 silver cars were recorded. What is the relative frequency of a silver car passing?	
8. Estimate the answer to: $253 \div 0.46$		18. Write down the 7^{th} term in the sequence given by: $T(n) = n^2 + 2n$			
9. Write down all the factors of 24		19. If $y = x^2 - x$, find the value of y when $x = -3$		25. ***Use π on the calculator*** Work out the volume of this cylinder? ***(Correct to 1decimal place)*** 	
10. Write down the HCF of 24 and 32		20. Write down the equation of a line parallel to the graph $y=2x – 4$			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

FORMING AND SOLVING KNOWLEDGE ORGANISER

EQUATIONS WITH UNKNOWNS ON BOTH SIDES AND BRACKETS

Solve $5x + 4 = 3x + 10$.

There are more xs on the left-hand side, so leave the equation as it is.

Subtract $3x$ from both sides. $2x + 4 = 10$

Subtract 4 from both sides. $2x = 6$

Divide both sides by 2. $x = 3$

Solve $3(2x + 5) + x = 2(2 - x) + 2$.

Multiply out both brackets. $6x + 15 + x = 4 - 2x + 2$

Simplify both sides. $7x + 15 = 6 - 2x$

There are more xs on the left-hand side, so leave the equation as it is.

Add $2x$ to both sides. $9x + 15 = 6$

Subtract 15 from both sides. $9x = -9$

Divide both sides by 9. $x = -1$

FORMING EQUATIONS FROM WORDS (INC SHAPES)

Equations are used to represent situations, so that you can solve real-life problems.

The rectangle shown has a perimeter of 40 cm.

Find the value of x.

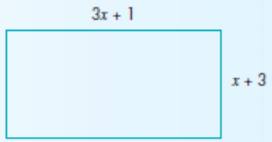
The perimeter of the rectangle is:

$3x + 1 + x + 3 + 3x + 1 + x + 3 = 40$

This simplifies to $8x + 8 = 40$.

Subtract 8. $8x = 32$

Divide by 8. $x = 4$



How to Solve a Simultaneous Equation Algebraically
Example A - EASY

$5x + y = 20$ (1) Label equations

$2x + y = 11$ (2) Subtract (2) from (1) to 'eliminate' x

$3x = 9$

$3x = 9$

$x = 3$

Substitute $x = 3$ into equation (1)

$5 \times 3 + y = 20$

$15 + y = 20$

$y = 5$

$x = 3$

$y = 5$

Are the solutions to my equations

How to Solve a Simultaneous Equation Algebraically
Example B – MORE CHALLENGING

$3x + 9y = 36$ (1)

$2x + 3y = 15$ (2) Label equations

$3x + 9y = 36$

$2x \times 3 + 3y \times 3 = 15 \times 3$ Multiply all terms by 3 to equate the coefficient of y, then you eliminate...

$3x + 9y = 36$

$6x + 9y = 45$

$-3x = -9$

$x = 3$ Subtract (2) from (1) to 'eliminate' y

Substitute $x = 3$ into equation (1)

$3 \times 3 + 9y = 36$

$9 + 9y = 36$

$9y = 27$

$y = 3$

$x = 3$

$y = 3$

Are the solutions to my equations

CHANGING THE SUBJECT

A formula usually has a single variable on one side of the equals sign. This is called the subject of the formula. Sometimes you will want to rearrange the formula so that one of the other variables becomes the subject. To do this you use inverse operations (in a similar way to solving equations) in order to isolate the new subject.

Example 1
Make r the subject of $C = 2\pi r$.

To isolate r, divide by 2π.

$$\frac{C}{2\pi} = r$$

We often write formulae with the subject on the left-hand side, so this becomes

$$r = \frac{C}{2\pi}$$

Example 2
Make x the subject of $y = \frac{x}{5} + 3$.

To isolate x, start by subtracting 3.

$$y - 3 = \frac{x}{5}$$

Next, multiply by 5 – remember to multiply each term of the left-hand side.

$$5(y - 3) = x$$

$$x = 5(y - 3)$$

Example 3
Make r the subject of $V = \frac{1}{3}\pi r^2 h$.

To start, isolate r^2 by multiplying by 3 and then dividing by πh.

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = r^2$$

Now we square root both sides.

$$\sqrt{\frac{3V}{\pi h}} = r$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

Example 4
Make x the subject of $3x + 5 = y - ax$.

When a formula contains the new subject more than once, start by isolating any terms including it on one side of the equals sign.

Here, add ax and subtract 5.

$$3x + ax = y - 5$$

Now we factorise the side with our new subject.

$$x(3 + a) = y - 5$$

Then divide by the bracket to leave x on its own.

$$x = \frac{y - 5}{3 + a}$$

Content

When we solve quadratic equations, there will be two

QUADRATIC EQUATIONS

You can also solve quadratic equations using a graph, find the points where the graph crosses the x-axis, these are your solutions.

Method 1 – Factorisation
First we factorise the quadratic into 2 brackets. As at least one of these brackets must equal 0. We then have 2 linear equations to solve.

Example
Solve $x^2 - 2x - 15 = 0$

Factorise into 2 brackets

$$(x + 3)(x - 5) = 0$$

Either

$$(x + 3) = 0 \text{ or } (x - 5) = 0$$

Therefore

$$x = -3 \text{ or } x = 5$$

Method 2 – Completing the Square
This method can be used when we can't easily factorise the quadratic and usually has surds in the solutions.

Example
Solve $x^2 - 4x - 3 = 0$

Rearrange so that the unknowns are on one side

$$x^2 - 4x = 3$$

Halve the coefficient of x. This number must be put into a bracket, along with x, and squared. We then subtract the square of this number

$$(x - 2)^2 - 4 = 3$$

Solve the equation

$$(x - 2)^2 = 7$$

$$x - 2 = \pm\sqrt{7}$$

$$x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

Remember that the square root of a number can be either positive or negative.

Method 3 – The Quadratic Formula
This method is also used when we cannot easily factorise the quadratic. This will be on a calculator paper and the answers will generally be decimals.

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For quadratic equations of the form $ax^2 + bx + c = 0$

Example
Solve $2x^2 + 11x - 5 = 0$. Give your answer to 2 decimal places.

$$a = 2, b = 11, c = -5$$

Substitute these into the quadratic formula, use brackets for negative numbers.

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

Put this into the calculator, first with a + and then with a – to find your two solutions.

$$x = -5.92 \text{ or } x = 0.42$$

ITERATION

- Iteration means the act of repeating a process with the aim of approaching a desired goal, target or result.

1) Re-arrange the equation to make one of the x's the subject
2) Make the subject x_{n+1} and the other x becomes x_n
3) Substitute in a value of x (this is either x_0 or x_1) to produce your first result
4) Now keep substituting your solutions back in until your answer converges on the required degree of accuracy required, or stipulated.

$$x^2 - 5x + 6 = 0$$

1) Re-arrange to make one of the x's the subject:

$$x^2 = 5x - 6$$
$$x = \sqrt{5x - 6}$$

2) Make the subject x_{n+1} and the other x becomes x_n

$$x_{n+1} = \sqrt{5x_n - 6}$$

3) Substitute in a value of x (this is either x_0 or x_1) to produce your first result

$$x_{n+1} = \sqrt{5x_n - 6}$$

- $x_1 = 4$
- $x_2 = \sqrt{5(4) - 6} = 3.741657\ldots$
- $x_3 = \sqrt{5(3.741657\ldots) - 6} = 3.564868\ldots$
- $x_4 = \sqrt{5(3.564868\ldots) - 6} = 3.438654\ldots$
- $x_5 = \sqrt{5(3.438654\ldots) - 6} = 3.345634\ldots$

...and so on. Carrying this on will eventually converge on one of the roots at $x = 3$

PROOF

'oof": **Verify** that..., **Show** that..., and **Prove** that....

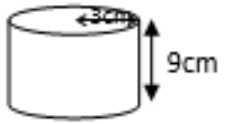
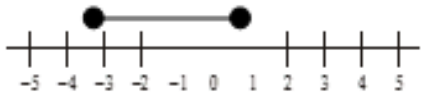
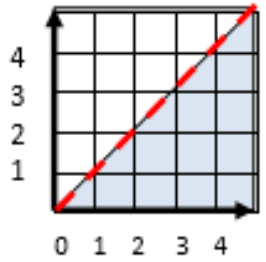
- At the lowest level (verification), all you have to do is to substitute numbers into the result to show that it works.
- At the middle level, you have to show that both sides of the result are the same algebraically.
- At the highest level (proof), you have to manipulate the left-hand side of the result to become its right-hand side.

The following example demonstrates these three different procedures.

THE DUSTON SCHOOL

Knowledge Recall **Term 4 HW: 3**

Date Due **Score to beat**

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. To decrease an amount by 4%, what single multiplier would you use?		11. Expand & simplify: $5(x-3)-2(2x+1)$		21. ***Use π on the calculator*** Work out the volume of this cylinder? *(Correct to 1 significant figure)* 	
2. Share £1000 in the ratio of 3 : 2		12. Give the inequality 			
3. Work out: $\frac{5}{8} \div \frac{2}{3}$		13. Work out the value of: 5x - 2y When x = -2 and y = -3		22. Sam ran at 6km/h for 2h 20min. What distance did he run?	
4. Round off 0.521 to one significant figure		14. Write down the nth term of this sequence: 5 11 21 35 ...			
5. Write down the LCM of 20 and 15		15. If $y = x^2 + 2x$, find the value of y when x = -2		23. 500 tickets are sold for a prize draw. The probability that Bill wins first prize is $\frac{1}{20}$. How many tickets did he buy?	
6. Write $0.\dot{5}$ as a fraction		16. Factorise: $p^2 - 1$			
7. The value of a bike depreciates by 55% per year. Work out the current value of a bike bought 2 years ago for £1300.		17. Multiply & simplify: (2a - 3)(2a + 1)		24. What inequality is represented here? 	
8. The cost of a phone has increased by 10% to £352. Work out the original price.		18. Make w the subject of the formula: $P=\frac{7w-10}{60}$			
9. Write 5×10^{-3} as an ordinary number		***Give your answer correct to 3sf*** 19. $A = \pi r^2 - \pi rs$ Find A when r = 2.7, s = 1.6		25. On a spinner: P(3) = ⅓ and the p(4) = ⅓ What is the probability of getting 3 or 4	
10. Work out $(8 \times 10^6) \times (9 \times 10^{-2})$ Give your answer in standard form		20. If $\cos 35^0 = \frac{8}{x}$, find x (3sf)			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	