THE DUSTON SCHOOL
Knowledge itself is power

Year 10 Maths Higher Knowledge Booklet Term 4

Name:

Class:

Order	Unit	Links	Pre-requisite skills
1	Integers, powers & roots		
2	Lines, angles & shape		
3	Simplifying & substituting	Unit 1	Using powers, listing factors, understanding product / sum.
4	Area and perimeter	Unit 2	Forming expressions for area/perimeter algebraically through use of brackets, correct notation and simplifying expressions.
5	Calculations & Accuracy	Unit 1	Understanding numbers.
6	Construction and LOCI	Unit 2	Measuring angles for bearings, parallel line angle facts.
7	FDP	Unit 1	Using powers, understanding lowest common multiples.
8	Sequences, functions and graphs	Unit 3/5	Substituting into a function applying BIDMAS to calculate coordinates, factorising for roots of quadratics, understanding powers and all 4 operations with negatives.
9	Ratio & Proportion	Unit 1/7	Decimals/powers as multipliers, calculating/understanding fractions as parts.
10	Transformations	Unit 2/8	Identifying 90/180/270 degrees, plotting mirror lines of basic functions.
11	Pythagoras and Trigonometry	Unit 1/2/3/4/5	Powers/surds, types of triangles, use in area/perimeter problems to find required lengths, rounding answers.
12	Forming and solving	Unit 3/4	Properties of 2d shapes, angle facts including polygons & parallel lines, algebraic notation and simplifying, forming expressions.
13	Measures	Unit 1/7	Calculating, multiplying decimals and powers of 10 for metric conversions.
14	Volume and Surface area	Unit 4/5/13	Area of 2d shapes, rounding/calculating with bounds, conversion of units (length/area/volume), calculating missing sides using pythagoras/ trigonometry.
15	Probability	Unit 1/7	Types of numbers, calculating with fractions & decimals.
16	Inequalities	Unit 12/8/5/7	Solving equations, rounding, plotting graphs for regions, calculating with fractions.
17	Statistics	Unit 1/6/9/16	Using a protractor for pie charts, proportion to calculate angles for a pie chart, use of inequality symbols for recording data.

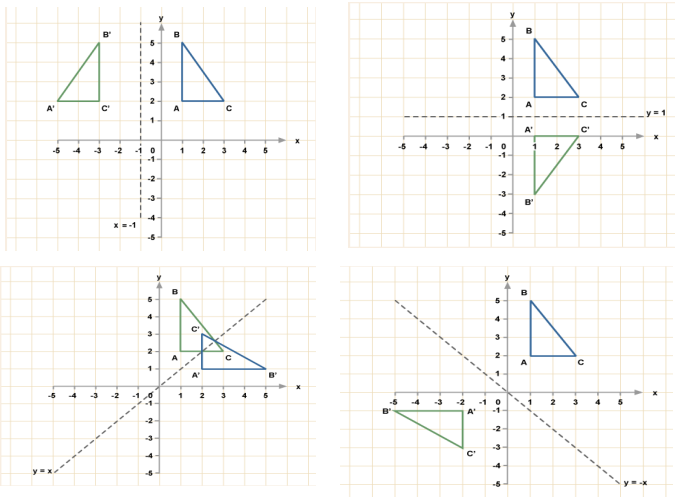
Homework 1 Due	
Homework 2 Due	
Homework 3 Due	

Year 10 - Term 4: Higher

Overview	Learning Objective		
Topic: Transformations **Big Questions** - Show me an example of one vector which is a scalar multiple of another. - What do you think an enlargement with a scale factor of –1/4 would look like?	- Describe all four transformations. - Combined transformations. (Rotations which is the same as an enlargement.) - Introduction to vectors. (Add, subtract and multiply vectors)	- Enlarge a shape by a negative scale factor given a centre. - Describe the changes and invariance achieved by combinations of rotations, reflections and transformations.	- Enlarge a shape by a negative fractional scale factor. - Vectors. - Understand the relationship between parallel vectors. - Vector proofs.
Topic: Pythagoras and Trigonometry **Big Questions** • Show me a question which can be solved using: - the sine rule. - the cosine rule. - ½ a b sin C - How does the mnemonic - SOHCAHTOA help you remember equations?	- Use Pythagoras Theorem to calculate the length of the hypotenuse fo a right angles triangle. - Use Pythagoras Theorem to calculate the length of any side of a right angled triangle. - Use Pythagoras Theorem to calculate the height of an isosceles triangle. - Use Pythagoras Theorem in practical problems -Find the distance between two coordinates.	- Know the exact values of sine, cosine and tangent at key angles (0, 30, 45, 60, 90 degrees). - SOHCAHTOA to calculate missing sides in right-angled triangles. - SOHCAHTOA to calculate missing angles in right-angled triangles. - Use SOHCAHTOA in practical problems. - Use the formula for area of a non-right-angled triangle.	- Use the sine rule to find missing sides and angles in non-right-angled triangles. - Use the cosine rule to find missing sides and angles in non-right-angled triangles. -Use Sine & Cosine combined in non-right angled triangles. - Sketch the graphs of: - y = sin x - y = cos x - y = tan x - Use Pythagoras' Theorem in 3D. -Use 3D trigonometry.
Topic: Forming and solving equations **Big Questions** - Prove algebraically that the sum of two consecutive odd numbers is even.	- Factorise and solve quadratics in the form $ax^2 + bx + c = 0$ where $a > 1$. -Rearrange formulae where the variable appears twice.	- Rearrange formulae that include brackets, fractions and square roots. - Solve quadratics using the quadratic formula. - Find approximate solutions to equations numerically using iteration. -Algebraic proof.	- Solve a pair of simultaneous equations where one is nonlinear. - Complete the square to solve quadratic equations. - Solve fractional quadratic equations (algebraic fractions) - Solve fractional linear equations with an unknown in the denominator.

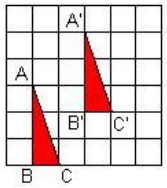
TRANSFORMATIONS KNOWLEDGE ORGANISER

REFLECTION



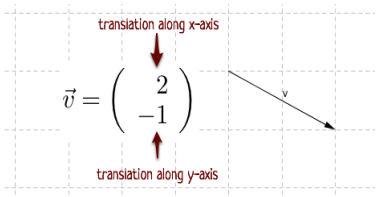
Name	Shape	Order of Rotational Symmetry
Parallelogram		2
Regular Polygon with n sides	Examples:	n
Rhombus		2
Circle		Unlimited
Trapezium		None
Kite		None

TRANSLATION



Each point moved 2 to the right and 2 up.

The vector describing this translation is: $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$



Equal vectors

If two vectors have the **same magnitude** and **direction**, then they are **equal**.

Adding vectors

Look at the graph below to see the movements between **PQ**, **QR** and **PR**.

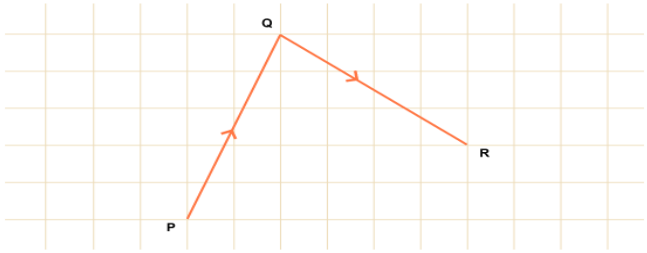
$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

Vector $\overrightarrow{PQ}$ followed by vector $\overrightarrow{QR}$ represents a movement from **P** to **R**.

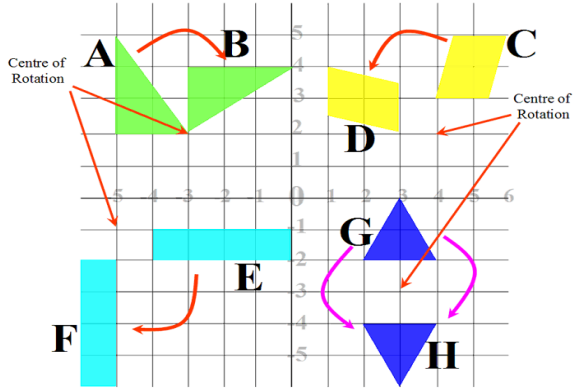
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

Written out the vector addition looks like this

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$



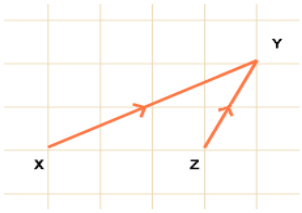
ROTATION



Subtracting vectors

Subtracting a vector is the same as adding a negative version of the vector (remember that making a vector negative means reversing its direction).

$\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$



Look at the diagram and imagine going from X to Z. How would you write the path in vectors using only the vectors $\overrightarrow{XY}$ and $\overrightarrow{ZY}$?

You could say it is vector $\overrightarrow{XY}$ followed by a backwards movement along $\overrightarrow{ZY}$.

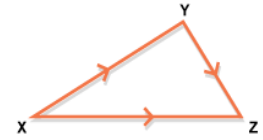
So we can write the path from X to Z as

$\overrightarrow{XY} - \overrightarrow{ZY} = \overrightarrow{XZ}$

Written out in numbers it looks like this:

$\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

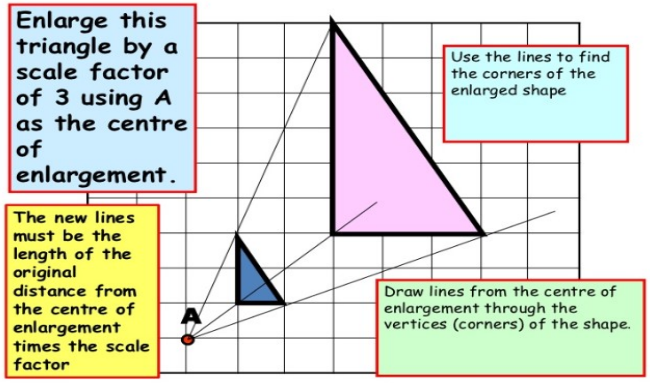
Resultant vectors

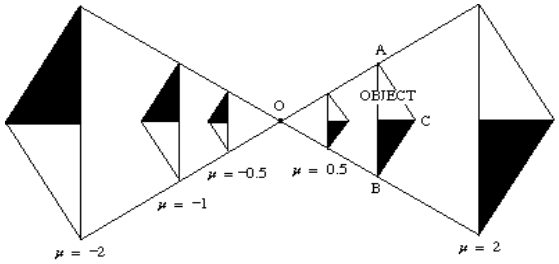


To travel from **X** to **Z**, it is possible to move along vector $\overrightarrow{XY}$ followed by $\overrightarrow{YZ}$. It is also possible to go directly along $\overrightarrow{XZ}$.

$\overrightarrow{XZ}$ is therefore known as the **resultant** of $\overrightarrow{XY}$ and $\overrightarrow{YZ}$.

ENLARGEMENT





Fractional scale factors

If we 'enlarge' a shape by a scale factor that is between -1 and 1, the image will be **smaller** than the object

Negative scale factors

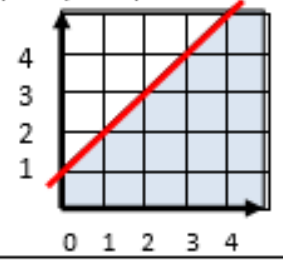
An enlargement using a negative scale factor is similar to an enlargement using a positive scale factor, but this time the image is on the other side of the centre of enlargement, and it is upside down.

THE DUSTON SCHOOL

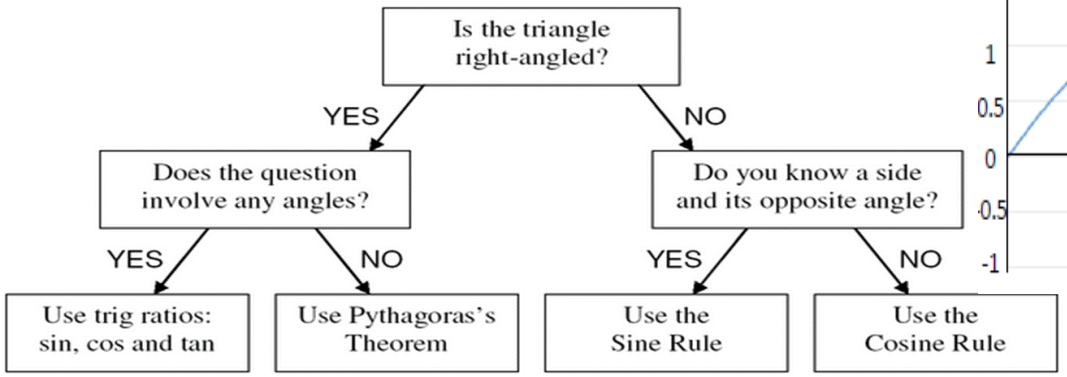
Knowledge Recall

Term4 HW: 1

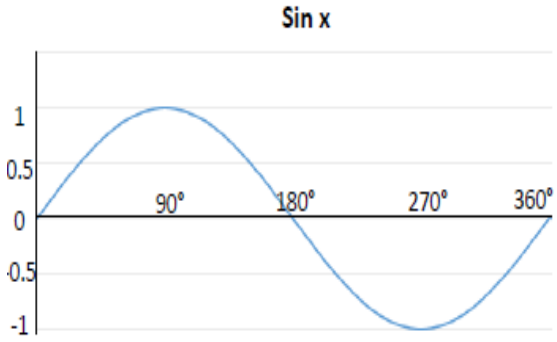
Date Due: _____ **Score to beat:** _____

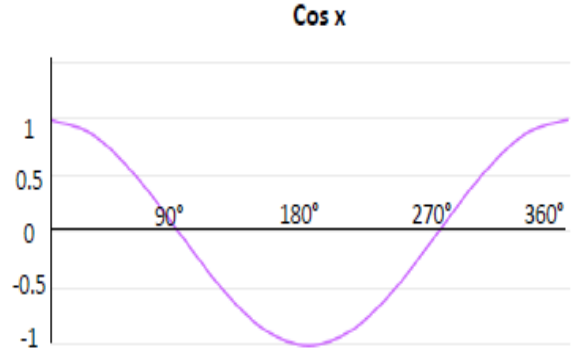
Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{4}{9}$ as a recurring decimal		11. Factorise: $a^2 - 9a + 20$		21. Linear-Quadratic-Cubic-Reciprocal Which type of graph is represented by this equation? $Y = 3-2x$	
2. Write $0.\dot{2}$ as a fraction		12. Factorise: $x^2 - y^2$			
3. Work out the balance for £600 invested for 3 years at 4.5% per annum		14. Multiply & simplify: $(3x + 2)(2x - 5)$		22. What inequality is represented here? 	
4. The value of a DS depreciates by 30% per year. Work out the current value of a DS bought 4 years ago for £99.		14. Multiply & simplify: $(a - 3b)^2$			
5. In a '60% off' sale, a dress was £26. Work out the original price.		15. Make r the subject of the formula: $S = 5r^2 + 7$		23. P(Jack is late to school any day) = 0.7 What is the probability that Jack will be late 2 days running?	
6. A computer has increased by 8% to £351. Work out the original price.		16. Make c the subject of the formula: $a^2 = b^2 + c^2$			
7. Write 0.00000834 in standard form:		17. $h = ut - \frac{1}{2}gt^2$ Find h when u = 100, t=1⅘ & g=6.4		24. Alf & Amy but tickets in a raffle P(Alf wins 1st prize) = 0.3 P(Amy wins 1st prize) = 0.25 What is the probability that Alf or Amy win 1st prize?	
8. Write 6.72×10^4 as an ordinary number		Give your answer correct to 3sf 18. $T = 2\pi\sqrt{\frac{l}{8}}$ Find T when l = 4⅘			
9. Work out $(7\times10^{-4}) \times (8 \times 10^{-3})$ Give your answer in standard form		19. If $\sin x^0 = \frac{7}{9}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the lower quartile reading 	
10. Work out $(5.63 \times 10^{-3}) - (4.28 \times 10^{-4})$ Give your answer in standard form		20. Each of these measures is rounded to 1dp: a = 8.3cm and b = 4.2cm Calculate the lower bound of a + b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

Pythagoras' and Trigonometry



BASIC Sine and Cosine Graphs





Cosine Rule

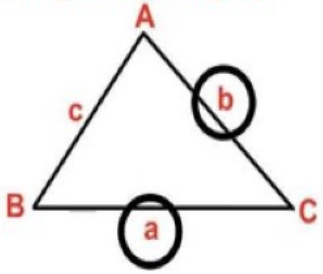
The Cosine Rule can be used in any triangle

$a^2 = b^2 + c^2 - 2bc \cos A$ Used to calculate an unknown **SIDE**

$\text{COS A} = \dfrac{b^2 + c^2 - a^2}{2bc}$ Used to calculate an unknown **ANGLE**

Area of a Triangle (Non Right-Angled Triangle)

$\text{Area} = \frac{1}{2}ab\,\text{Sin}C$



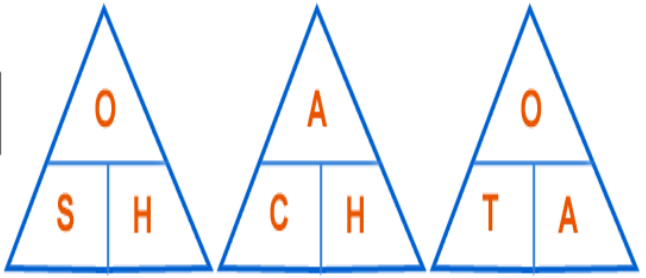
This formula finds the area of a non right-angled triangle from 2 sides

Key Facts - Exact Trigonometric Values

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$

Key Facts – Pythagoras' Theorem

$a^2 + b^2 = c^2$	Where C is the hypotenuse
$c^2 - a^2 = b^2$	Where the unknown side is the shorter side (aka 'leg')



Sine Rule

The Sine Rule can be used in any triangle

Used to calculate an unknown **SIDE**

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

Used to calculate an unknown **ANGLE**

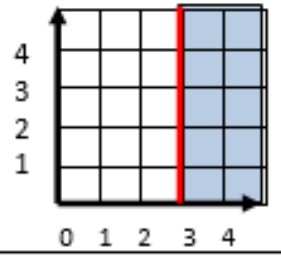
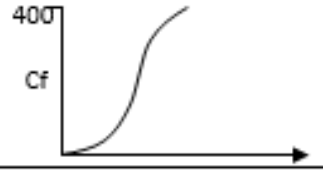
$$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

THE DUSTON SCHOOL

Knowledge Recall

Term4 HW: 2

Date Due: _____ **Score to beat:** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{7}{15}$ as a recurring decimal		11. Factorise: $a^2 - 2a - 15$		21. Linear-Quadratic-Cubic-Reciprocal Which type of graph is represented by this equation? $y = \frac{10}{x}$	
2. Write $0.\dot{7}$ as a fraction		12. Factorise: $4x^2 - 9y^2$			
3. Work out the balance for £600 invested for 5 years at 7.5% per annum		14. Multiply & simplify: $(3x - 2)(2x - 5)$		22. What inequality is represented here? 	
4. The value of a mobile depreciates by 40% per year. Work out the current value of a mobile bought 3 years ago for £124.		14. Multiply & simplify: $(2a + 3)^2$			
5. In a '60% off' sale, an outfit was £86. Work out the original price.		15. Make r the subject of the formula: $S = r^2 - 2t$		23. P(Jack is late to school any day) = 0.1 What is the probability that Jack will be late 2 days running?	
6. A fuel bill has increased by 16% to £139.20. Work out the original cost.		16. Make b the subject of the formula: $a^2 = b^2 - c^2$			
7. Write 280 in standard form:		17. $v = \sqrt{u^2 + 2as}$ Find v when u = 20 a=6 & s=52		24. Alf & Amy but tickets in a raffle P(Alf wins 1st prize) = 0.7 P(Amy wins 1st prize) = 0.12 What is the probability that Alf or Amy win 1st prize?	
8. Write 5.6×10^{-4} as an ordinary number		Give your answer correct to 3sf 18. $v = \sqrt{u^2 + 2as}$ Find v when u = 2.4 a=3.2 & s=5.25			
9. Work out $(5 \times 10^{-5}) \times (2 \times 10^4)$ Give your answer in standard form		19. If $\tan x^0 = \frac{12}{5}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the upper quartile reading 	
10. Work out $(6.72 \times 10^{-3}) + (2.84 \times 10^{-5})$ Give your answer in standard form		20. Each of these measures is rounded to 1dp: a = 8.3cm and b = 4.2cm Calculate the upper bound of a - b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	

FORMING AND SOLVING KNOWLEDGE ORGANISER

EQUATIONS WITH UNKNOWNS ON BOTH SIDES AND BRACKETS

Solve $5x + 4 = 3x + 10$.

There are more xs on the left-hand side, so leave the equation as it is.

Subtract $3x$ from both sides. $2x + 4 = 10$

Subtract 4 from both sides. $2x = 6$

Divide both sides by 2. $x = 3$

Solve $3(2x + 5) + x = 2(2 - x) + 2$.

Multiply out both brackets. $6x + 15 + x = 4 - 2x + 2$

Simplify both sides. $7x + 15 = 6 - 2x$

There are more xs on the left-hand side, so leave the equation as it is.

Add $2x$ to both sides. $9x + 15 = 6$

Subtract 15 from both sides. $9x = -9$

Divide both sides by 9. $x = -1$

FORMING EQUATIONS FROM WORDS (INC SHAPES)

Equations are used to represent situations, so that you can solve real-life problems.

The rectangle shown has a perimeter of 40 cm.

Find the value of x.

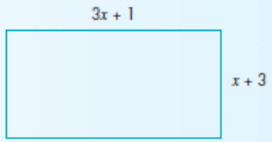
The perimeter of the rectangle is:

$3x + 1 + x + 3 + 3x + 1 + x + 3 = 40$

This simplifies to $8x + 8 = 40$.

Subtract 8. $8x = 32$

Divide by 8. $x = 4$



How to Solve a Simultaneous Equation Algebraically
Example A - EASY

$5x + y = 20$ (1) Label equations

$2x + y = 11$ (2) Subtract (2) from (1) to 'eliminate' x

$3x = 9$

$3x = 9$

$x = 3$

Substitute $x = 3$ into equation (1)

$5 \times 3 + y = 20$

$15 + y = 20$

$y = 5$

$x = 3$

$y = 5$

Are the solutions to my equations

How to Solve a Simultaneous Equation Algebraically
Example B – MORE CHALLENGING

$3x + 9y = 36$ (1)

$2x + 3y = 15$ (2) Label equations

$3x + 9y = 36$

$2x \times 3 + 3y \times 3 = 15 \times 3$ Multiply all terms by 3 to equate the coefficient of y, then you eliminate...

$3x + 9y = 36$

$6x + 9y = 45$

$-3x = -9$

$x = 3$ Subtract (2) from (1) to 'eliminate' y

Substitute $x = 3$ into equation (1)

$3 \times 3 + 9y = 36$

$9 + 9y = 36$

$9y = 27$

$y = 3$

$x = 3$

$y = 3$

Are the solutions to my equations

CHANGING THE SUBJECT

A formula usually has a single variable on one side of the equals sign. This is called the subject of the formula. Sometimes you will want to rearrange the formula so that one of the other variables becomes the subject. To do this you use inverse operations (in a similar way to solving equations) in order to isolate the new subject.

Example 1
Make r the subject of $C = 2\pi r$.

To isolate r, divide by 2π.

$$\frac{C}{2\pi} = r$$

We often write formulae with the subject on the left-hand side, so this becomes

$$r = \frac{C}{2\pi}$$

Example 2
Make x the subject of $y = \frac{x}{5} + 3$.

To isolate x, start by subtracting 3.

$$y - 3 = \frac{x}{5}$$

Next, multiply by 5 – remember to multiply each term of the left-hand side.

$$5(y - 3) = x$$

$$x = 5(y - 3)$$

Example 3
Make r the subject of $V = \frac{1}{3}\pi r^2 h$.

To start, isolate r^2 by multiplying by 3 and then dividing by πh.

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = r^2$$

Now we square root both sides.

$$\sqrt{\frac{3V}{\pi h}} = r$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

Example 4
Make x the subject of $3x + 5 = y - ax$.

When a formula contains the new subject more than once, start by isolating any terms including it on one side of the equals sign.

Here, add ax and subtract 5.

$$3x + ax = y - 5$$

Now we factorise the side with our new subject.

$$x(3 + a) = y - 5$$

Then divide by the bracket to leave x on its own.

$$x = \frac{y - 5}{3 + a}$$

Content

When we solve quadratic equations, there will be two

QUADRATIC EQUATIONS

You can also solve quadratic equations using a graph, find the points where the graph crosses the x-axis, these are your solutions.

Method 1 – Factorisation
First we factorise the quadratic into 2 brackets. As at least one of these brackets must equal 0. We then have 2 linear equations to solve.

Example
Solve $x^2 - 2x - 15 = 0$

Factorise into 2 brackets

$$(x + 3)(x - 5) = 0$$

Either

$$(x + 3) = 0 \text{ or } (x - 5) = 0$$

Therefore

$$x = -3 \text{ or } x = 5$$

Method 2 – Completing the Square
This method can be used when we can't easily factorise the quadratic and usually has surds in the solutions.

Example
Solve $x^2 - 4x - 3 = 0$

Rearrange so that the unknowns are on one side

$$x^2 - 4x = 3$$

Halve the coefficient of x. This number must be put into a bracket, along with x, and squared. We then subtract the square of this number

$$(x - 2)^2 - 4 = 3$$

Solve the equation

$$(x - 2)^2 = 7$$

$$x - 2 = \pm\sqrt{7}$$

$$x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

Remember that the square root of a number can be either positive or negative.

Method 3 – The Quadratic Formula
This method is also used when we cannot easily factorise the quadratic. This will be on a calculator paper and the answers will generally be decimals.

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For quadratic equations of the form $ax^2 + bx + c = 0$

Example
Solve $2x^2 + 11x - 5 = 0$. Give your answer to 2 decimal places.

$$a = 2, b = 11, c = -5$$

Substitute these into the quadratic formula, use brackets for negative numbers.

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

Put this into the calculator, first with a + and then with a – to find your two solutions.

$$x = -5.92 \text{ or } x = 0.42$$

ITERATION

- Iteration means the act of repeating a process with the aim of approaching a desired goal, target or result.

1) Re-arrange the equation to make one of the x's the subject
2) Make the subject x_{n+1} and the other x becomes x_n
3) Substitute in a value of x (this is either x_0 or x_1) to produce your first result
4) Now keep substituting your solutions back in until your answer converges on the required degree of accuracy required, or stipulated.

$$x^2 - 5x + 6 = 0$$

1) Re-arrange to make one of the x's the subject:

$$x^2 = 5x - 6$$
$$x = \sqrt{5x - 6}$$

2) Make the subject x_{n+1} and the other x becomes x_n

$$x_{n+1} = \sqrt{5x_n - 6}$$

3) Substitute in a value of x (this is either x_0 or x_1) to produce your first result

$$x_{n+1} = \sqrt{5x_n - 6}$$

- $x_1 = 4$
- $x_2 = \sqrt{5(4) - 6} = 3.741657\ldots$
- $x_3 = \sqrt{5(3.741657\ldots) - 6} = 3.564868\ldots$
- $x_4 = \sqrt{5(3.564868\ldots) - 6} = 3.438654\ldots$
- $x_5 = \sqrt{5(3.438654\ldots) - 6} = 3.345634\ldots$

...and so on. Carrying this on will eventually converge on one of the roots at $x = 3$

PROOF

'oof": **Verify** that..., **Show** that..., and **Prove** that....

- At the lowest level (verification), all you have to do is to substitute numbers into the result to show that it works.
- At the middle level, you have to show that both sides of the result are the same algebraically.
- At the highest level (proof), you have to manipulate the left-hand side of the result to become its right-hand side.

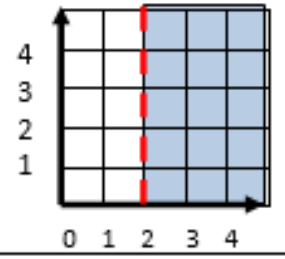
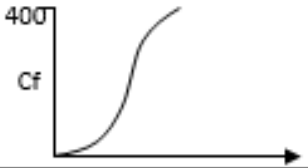
The following example demonstrates these three different procedures.

THE DUSTON SCHOOL

Knowledge Recall

Term4 HW: 3

Date Due: _____ **Score to beat:** _____

Section A:Number		Section B: Algebra Geometry & measures		Section C: Using and applying	
1. Write $\frac{11}{15}$ as a recurring decimal		11. Factorise: $x^2 + 2x + 1$		21. Linear-Quadratic-Cubic-Reciprocal Which type of graph is represented by this equation? $Y=2x^3 -5x^2$	
2. Write $0.\dot{5}$ as a fraction		12. Factorise: $x^2 - 16y^2$			
3. Work out the balance for £1500 invested for 3 years at 6.5% per annum		14. Multiply & simplify: $(x + 2)(5x - 3)$		22. What inequality is represented here? 	
4. The value of a mobile depreciates by 40% per year. Work out the current value of a mobile bought 3 years ago for £225.		14. Multiply & simplify: $(2a - 3)^2$			
5. In a '60% off' sale, an outfit was £144. Work out the original price.		15. Make r the subject of the formula: $S= r^2 - t^2$		23. P(Jack is late to school any day) = 0.6 What is the probability that Jack will be late 2 days running?	
6. A fuel bill has increased by 18% to £141.60. Work out the original cost.		16. Make c the subject of the formula: $a^2 = b^2 - c^2$			
7. Write 0.056 in standard form:		17. $v=\sqrt{u^2 + 2as}$ Find v when u = 16 a=8 & s=33		24. Alf & Amy but tickets in a raffle P(Alf wins 1st prize) = 0.28 P(Amy wins 1st prize) = 0.02 What is the probability that Alf or Amy win 1st prize?	
8. Write 4.651×10^6 as an ordinary number		Give your answer correct to 3sf 18. $v=\sqrt{u^2 + 2as}$ Find v when u = 9.1 a=-4.7 & s=3.04			
9. Work out $(4\times10^3) + (6 \times 10^4)$ Give your answer in standard form		19. If $\tan 18^0 = \frac{x}{12}$, find x (3sf)		25. Show on the cumulative frequency graph how to take the inter-quartile range reading 	
10. Work out $(4.32 \times10^{-3})-(4.28 \times 10^{-5})$ Give your answer in standard form		20. Each of these measures is rounded to 1dp: a = 8.3cm and b = 4.2cm Calculate the lower bound of a - b			
Total (A)		Total (B)		Total (C)	
Test Total (A+B+C)		R (0-9)	Y (10-19)	G (20-25)	