THE DUSTON SCHOOL
Knowledge itself is power
TDS 4-19

Year 11 Foundation Maths Knowledge Organiser Term 1

Name:	Class:

Topics This term:

- Probability – GCSE Mathematics
  - Venn Diagrams
  - Relative Frequency
  - Tree Diagrams
- Time Series – GCSE Statistics
  - Trend Lines
  - Moving Averages
- Number and Algebra – GCSE Mathematics
  - Re-cap and strengthening the Number and Algebra Skills

RESPECT

In Mathematics, a classroom environment should always be respectful. Students can show respect through:

- **Supporting each other with their learning.** Pupils should recognise that every individual has their own strengths and weaknesses and, as a class, we should 'up-lift' students.
- **Students should not be felt to be rushed by others in the classroom**. Respect that all students have different experiences and therefore will access the knowledge at different rates.
- **Being Polite.** As no different to the rest of school. Students should embrace diversity and treat all others with tolerance and decency.

ASPIRATION

- **Building logical processes.** Understanding that learning mathematical concepts improves our logical reasoning which improves other aspects of our lives: language, culture, games etc. the essence of mathematics is in respect of ideas, structures and relationships by logical reasoning.
- **Every day needs.** Understanding that being numerate, along with literate, is a strong indicator of long-term success and students' ability to climb the tree of knowledge.

RESILIENCE

- **I don't know it… yet.** Understanding that maths can be abstract and that, as with anything new, it will take time to learn. With time, you will succeed.
- **Mathematical concept won't always come easily.** Understanding that getting things wrong is a frustrating and not pleasant feeling but, to succeed, it is a passage we need to go through.
- **Practice makes permanent.** Mathematics is a logical subject such that, rehearsal and repetition of method is the key to being successful and committing the knowledge to long-term memory. This process takes time and will come with failures along the way which we must persevere through.

Foundation Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \frac{1}{2}(a + b)\,h$

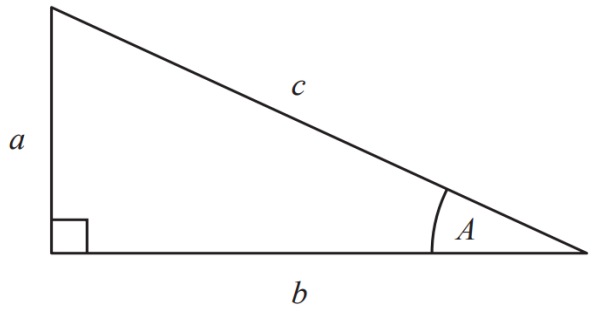
Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle $= 2\pi r = \pi d$

Area of a circle $= \pi r^2$

Pythagoras' Theorem and Trigonometry



In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued $= P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

END OF EXAM AID

MathsWatch

Grade 1	
Clip	Topic
1	Place Value
2	Ordering Integers
3	Ordering Decimals
4	Reading Scales
5	Simple Mathematical Notation
6a	Real-Life Tables - Time
6b	Real-Life Tables - Timetables and Distance Tables
7	Introduction to Algebraic Conventions
8	Coordinates
9	Simple Geometric Definitions
10	Polygons
11	Symmetries
12a	Tessellations
12b	Congruent Shapes
13	Names of Angles
14	The Probability Scale
15	Tally Charts and Bar Charts
16	Pictograms

Grade 2	
Clips	Topics
17	Adding Integers and Decimals
18	Subtracting Integers and Decimals
19	Multiplying Integers
20	Dividing Integers
21	Inverse Operations
22a	Money Questions - Non-Calculator Questions
22b	Money Questions - Calculator Questions
23	Negatives in Real Life
24	Introduction to Fractions
25	Equivalent Fractions
26	Simplifying Fractions
27	Half-Way Values
28	Factors, Multiples and Primes
29	Introduction to Powers/Indices
30	Multiplying and Dividing by Powers of 10
31	Rounding to the Nearest 10, 100, 1000
32	Rounding to Decimal places
33	Simplifying - Addition and Subtraction
34	Simplifying - Multiplication
35	Simplifying - Division
36	Function Machines
37	Generating a Sequence - Term to Term
38	Introduction to Ratio
39	Using Ratio for Recipe Questions
40	Introduction to Percentages
41	Value for Money
42	Introduction to Proportion
43	Properties of Solids
44	Nets
45	Angles on a Line and at a Point
46a	Measuring and drawing Angles - Measuring
46b	Measuring and drawing Angles - Drawing
47	Drawing a Triangle Using a Protractor
48	Reflections
49	Rotations
50	Translations
51	Plans and Elevations
52	Perimeters
53	Area of a Rectangle
54	Area of a Triangle
55	Area of a Parallelogram
56	Area of a Trapezium
57	Frequency Trees
58	Listing Outcomes
59	Calculating Probabilities
60	Mutually Exclusive Events
61	Two-Way Tables
62	Averages and the Range
63	Data - Discrete and Continuous
64	Vertical Line Charts
65	Frequency Tables and Diagrams

Grade 3	
Clip	Topic
66	Multiplying Decimals
67	Dividing Decimals
68a	Four Rules of Negatives - Adding and Subtracting
68b	Four Rules of Negatives - Multiplying and Dividing
69	Listing Strategies
70	Comparing Fractions
71a	Adding and Subtracting Fractions
71b	Adding and Subtracting Fractions
72	Finding a Fraction of an Amount
73	Multiplying Fractions
74	Dividing Fractions
75	BODMAS/BIDMAS
76	Reciprocals
77	Calculator Questions
78	Product of Primes
79	Highest Common Factor (HCF)
80	Lowest Common Multiple (LCM)
81	Squares, Cubes and Roots
82	Working with Indices
83	Standard Form
84	Decimals and Fractions
85	Fractions, Percentages, Decimals
86	Percentage of an Amount (Calc.)
87	Percentage of an Amount (Non-Calc.)
88	Change to a Percentage (Calc.)
89	Change to a Percentage (Non-Calc.)
90	Rounding to Significant Figures
91	Estimating Answers
92	Using Place Value
93	Expanding Brackets
94	Simple Factorisation
95	Substitution
96	Straight Line Graphs
97	The Gradient of a Line
98	Drawing Quadratic Graphs
99	Sketching Functions
100	Solving Equations using Flowcharts
101	Subject of a Formula using Flowcharts
102	Generating a Sequence from the nth Term
103	Finding the nth Term
104	Special Sequences
105	Exchanging Money
106	Sharing using Ratio
107	Ratios, Fractions and Graphs
108	Increase/Decrease by a Percentage
109	Percentage Change
110	Reverse Percentage Problems
111	Simple Interest
112	Metric conversions
113	Problems on Coordinate Axes
114a	Surface Area of a Prism - Cuboids
114b	Surface Area of a Prism - Triangular Prisms
115	Volume of a Cuboid
116	Circle Definitions
117	Area of a Circle
118	Circumference of a Circle
119	Volume of a Prism
120	Angles and Parallel Lines
121	Angles in a Triangle
122	Properties of Special Triangles
123	Angle Sum of Polygons
124	Bearings
125	Experimental Probabilities
126	Possibility Spaces
127a	Venn Diagrams - Introduction
127b	Venn Diagrams - Notation
128a	Representing Data - Pie Charts
128b	Representing Data - Stem and Leaf Diagrams
129	Scatter Diagrams
130a	Averages from a table - Basics
130b	Averages from a table - Estimate for the Mean

Grade 4	
Clip	Topic
131	Index Notation
132	Introduction to Bounds
133	Midpoint of a Line on a Graph
134a	Expanding and Simplifying Brackets - Single
134b	Expanding and Simplifying Brackets - Double
135a	Solving Equations - Balancing
135b	Solving Equations - Float & Ping
136	Rearranging Simple Formulae
137	Forming Formulae and Equations
138	Inequalities on a Number Line
139	Solve Linear Inequalities
140	Simultaneous Equations Graphically
141	Fibonacci Sequences
142	Compound Units
143	Distance-Time Graphs
144	Similar Shapes
145a	Constructing Perpendiculars - Bisecting a Line
145b	Constructing Perpendiculars - From any Point
145c	Bisecting an Angle
146	Loci
147	Draw a Triangle Using Compasses
148	Enlargements
149	Tangents, Arcs, Sectors and Segments
150a	Pythagoras' Theorem - A Simple Approach
150b	Pythagoras' Theorem - An Algebraic Approach
150c	Pythagoras' Theorem - Line on a Graph
151	Simple Tree Diagrams
152	Sampling Populations
153	Time Series

Grade 5	
Clips	Topic
154	Negative Indices
155	Error Intervals
156	Mathematical Reasoning
157	Factorising and Solving Quadratics
158	The Difference of Two Squares
159a	Equation of a Straight Line - y=mx+c
159b	Equation of a Straight Line - Gradient
160	Roots and Turning Points of Quadratics
161	Cubic and Reciprocal Graphs
162	Simultaneous Equations Algebraically
163	Geometric Progressions
164	Compound Interest and Depreciation
165a	Ratio Questions - Standard Questions
165b	Ratio Questions - Questions with Overlap
165c	Ratio Questions - Ratios, Fractions, Equations
166	Congruent triangles
167	Sectors of a Circle
168	Trigonometry
169	Spheres
170	Pyramids
171	Cones
172	Frustums
173	Exact Trigonometric Values
174	Introduction to Vectors
175	Harder Tree Diagrams
176	Stratified sampling

UNIT 5 : Time Series

Time Series: a statistical diagram used to represent data over a period of ***time*** where time is always plotted on the x axis

Points are plotted and then joined like a dot-to-dot

Moving averages are an efficient and practical way of finding the trend and mean you make more **ACCURATE** predictions

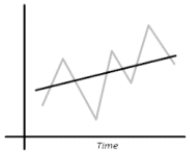
Moving averages tend to reduce the amount of **VARIABILITY** present in a time series

An 'n-point' moving average relies on how many **SEASONS** are repeated

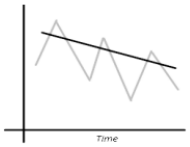
To calculate a moving average, we must work out the **MEAN** of each n points in the data, overlapping values so the averages 'move'

UNIT 5 : Time Series

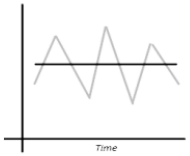
The general trend can be shown on the graph by drawing a trend line. Look at the general trend of the data and draw a line through the middle of the data



- This is an example of a **'rising trend'**
- It can also be called an **'upwards trend'**
- It can also be called an **'increasing trend'**
- We must NEVER use the term **'positive'**



- This is an example of a **'falling trend'**
- It can also be called an **'downwards trend'**
- It can also be called an **'decreasing trend'**
- We must NEVER use the term **'negative'**



- This is an example of a **'level trend'**
- We must NEVER use the term **'no trend'**

UNIT 5 : Time Series

The equation of the trend line

$y = mx + c$ or $y = a + bx$

To find the gradient you need to:

- pick 2 coordinates on your line
- divide the difference in the y coordinates, by the difference in the x coordinates

N.B. It is important to pay close attention to the scale on both the x and the y axis and whether the trend line is going up (positive gradient) or going down (negative gradient)

The gradient tells us how much y increases each time x increases by 1
Think about what this means in context of the question by considering what the scale of the x axis is.

UNIT 5 : Time Series

Seasonal Variation: when data follows the same pattern each season over a series of time frames.
Seasonal variation can help us to make more accurate predictions for future values

GCSE Mathematics Foundation Tier

Number · Algebra · Ratio, proportion and rates of change · Geometry & measures · Probability · Statistics

BIDMAS N3

...or BODMAS. Use the correct order of operations; take care when using a calculator.

- **B**rackets
- **I**ndices (or p**O**wers)
- **D**ivision and **M**ultiplication
- **A**ddition and **S**ubtraction

Types of number N4

Integer: a "whole" number

Factors; the divisors of an integer

→ **Factors of 12 are 1, 2, 3 ,4, 6, 12**

Multiples; a "times table" for an integer (will continue indefinitely)

→ **Multiples of 12 are 12, 24, 36 ...**

Prime number: an integer which has exactly two factors (1 and the number itself). Note: 1 is not a prime number.

2, 3, 5, 7, 11, 13, 17, 19

HCF, LCM N4

Highest **C**ommon **F**actor (HCF)

→ **Factors of 6 are 1, 2, 3, 6**
Factors of 9 are 1, 3, 9
HCF of 6 and 9 is 3

Lowest **C**ommon **M**ultiple (LCM)

→ **Multiples of 6 are 6, 12, 18, 24, ...**
Multiples of 9 are 9, 18, 27, 36, ...
LCM of 6 and 9 is 18

Prime factors N4

Write a number as a product of its prime factors; use indices for repeated factors:

→ $720 = 5 \times 3^2 \times 2^4$

Powers and roots N6, N7

Special indices: for any value a:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

→ $$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

Calculating with fractions N8

Adding or subtracting fractions; use a common denominator...

→ $$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

Multiplying fractions; multiply numerators and denominators...

→ $$\frac{4}{7} \times \frac{2}{3} = \frac{8}{21}$$

Dividing fractions; "flip" the second fraction, then multiply...

→ $$\frac{2}{7} \div \frac{5}{6} = \frac{2}{7} \times \frac{6}{5} = \frac{12}{35}$$

Fractions, decimals N10

Fraction is numerator ÷ denominator

→ $$\frac{5}{8} = 5 \div 8 = 0.625$$

Use place values to change decimals to fractions. Simplify where possible.

→ $$0.45 = \frac{45}{100} = \frac{9}{20}$$

Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{4}$
0.5	0.25	0.1	0.2	0.75

Percentages N8

10% - divide by 10
5% - divide by 10, then divide by 2
1% - divide by 10, then divide by 10 again

Standard form N9

Standard form numbers are of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Standard units N13

1 tonne = 1 000 kilograms
1 kilogram = 1 000 grams

1 kilometre = 1 000 metres
1 metre = 100 centimetres
= 1 000 millimetres
1 centimetre = 10 millimetres

1 day = 24 hours
1 hour = 60 minutes = 3 600 seconds
1 minute = 60 seconds

Rounding N15

Truncate the number, then use a "decider digit" to round up or down.

Decimal places: use the decimal point

→ **162.3681 to 2dp;**
162.36 | 81 = **162.37 to 2dp**

Significant figures: use the first non-zero digit.

→ **162.3681 to 2sf;**
16 | 2.3681 = **160 to 2sf**

→ **0.007 039 to 3sf;**
0.007 03 | 9 = **0.007 04 to 3sf**

Error intervals N15

Find the range of numbers that will round to a given value:

→ **$x = 5.83$ (2 decimal places)**
$5.825 \le x < 5.835$

→ **$y = 46$ (2 significant figures)**
$45.5 \le y < 46.5$

Note use of ≤ and <, and that the last significant figure of each is 5.

Algebraic notation A1

$$ab = a \times b$$
$$3y = y + y + y$$
$$a^2 = a \times a$$
$$a^3 = a \times a \times a$$
$$a^2b = a \times a \times b$$
$$\frac{a}{b} = a \div b$$

Equations and identities A3

An equation is true for some particular value of x...

→ **$2x + 1 = 7$ is true if $x = 3$**

...but an identity is true for every value of x

→ **$(x + a)^2 \equiv x^2 + 2ax + a^2$**

(note the use of the symbol ≡)

Laws of indices A4

For any value a:

$$a^x \times a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$

→ $$\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6} \text{ or } 8q^9p^{-6}$$

Sequences A24, A25

Triangular numbers:

1st	2nd	3rd	4th	5th
1	3	6	10	15

Square numbers ($n^2 = n \times n$):

1^2	2^2	3^2	4^2	5^2
1	4	9	16	25

Cube numbers ($n^3 = n \times n \times n$):

1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

nth term of an arithmetic (linear) sequence is $an + d$

→ **nth term of 5,8,11,14,... is $3n+2$ (always increases by 3; first term is $3 \times 1 + 2 = 5$)**

Geometric sequence; multiply each term by a constant ratio

→ **3, 6, 12, 24, ... (ratio is 2)**

Fibonacci sequence; make the next term by adding the previous two ...

→ **2, 4, 6, 10, 16, 26, 42, ...**

Probability P8, P9

$$p = \frac{n(\text{equally likely favourable outcomes})}{n(\text{equally likely possible outcomes})}$$

$p = 0$	impossible
$0 < p < 0.5$	unlikely
$p = 0.5$	evens
$0.5 < p < 1$	likely
$p = 1$	certain

Probability rules P8, P9

Multiply for independent events

→ **P(6 on dice and H on coin)**

$$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

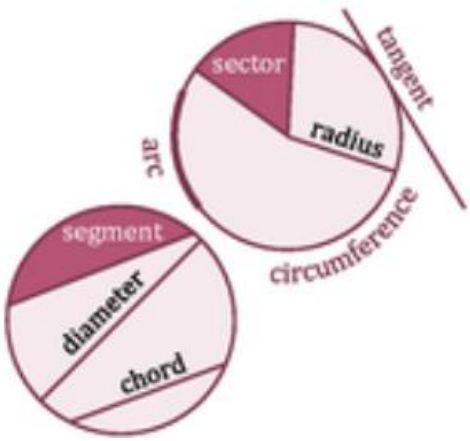
Add for mutually exclusive events

→ **P(5 or 6 on dice)**

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

Apply these rules to tree diagrams.

Parts of a circle G9



Division using ratio R5

Use a ratio for unequal sharing

→ **Divide £480 in the ratio 7 : 5**
7 + 5 = 12, then £480 ÷ 12 = £40
7 × £40 = £280, 5 × £40 = £200
(check: £280 + £200 = £480 ✓)

Ratio and fractions R8

Link between ratios and fractions

→ **Boys to girls in ratio 2 : 3**
$\frac{2}{5}$ are boys, $\frac{3}{5}$ are girls.

Percentages R9

$$y \text{ percent of } x = \frac{y}{100} \times x$$

→ **Increase £58 by 26%.**

$$\frac{26}{100} \times £58 = £15.08$$

£58 + £15.08 = £73.08

$$y \text{ as a percentage of } x = \frac{y}{x} \times 100\%$$

→ **The population of a town increase s from 3 500 to4 620. Fi nd the percentage increase.**

$$\frac{1\,120}{3\,500} \times 100\% = 32\%$$

Note: fraction = $\frac{\text{increase}}{\text{original}}$

Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{100}$
50%	25%	10%	20%	1%

Compound Measures R11

→ **A car travels90 miles in1 hour, 30 minutes. Find its average speed.**
90 miles ÷ 1.5 hours = 60 mph

Averages & Range S4

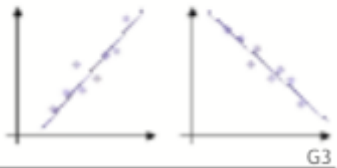
Mode: most frequently occurring

Median: put the data in numerical order, then choose the middle one

$$\text{Mean} = \frac{\text{total of items of data}}{\text{number of items of data}}$$

Range: Biggest number take smallest

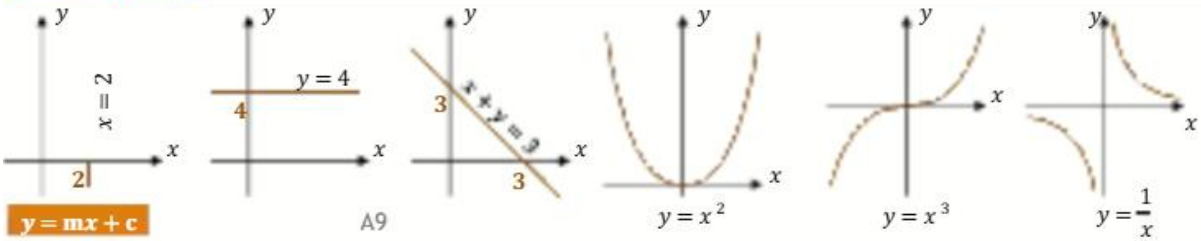
Correlation S6



G3

Here is pretty much all the Foundation Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An ➔ points to an illustrative example. The codes refer to the DfE subject content. Pin this to a wall, keep it on your desk, carry it in your bag, make no tes on it (sorry, don't take it into the examination)...

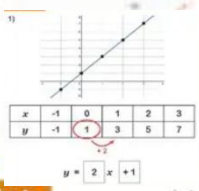
Standard graphs A12



y = mx + c A9

Equation of straight line $y = mx + c$
m is the gradient; c is the y intercept:

To plot a straight line, create a table of values, plot the co-ordinates, and join with a straight line



Expanding brackets A4

$$p(q+r) = pq + pr$$

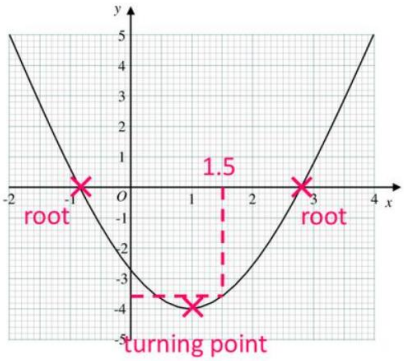
➔ $5(x-2y) = 5x - 10y$

$$(x+a)(x+b) = x^2 + ax + bx + ab$$

➔ $(2x-3)(x+5)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$

Reverse of expanding is factorising - putting an expression into brackets.

Quadratics A18



Difference of two squares A4

$$a^2 - b^2 = (a+b)(a-b)$$

➔ $x^2 - 25 = (x+5)(x-5)$

Simultaneous equations A19

➔ Solve $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$

Multiply to match a term in x or y

$$\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$$

Add or subtract to cancel...

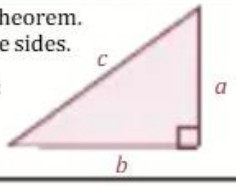
$19x = 76$, so $x = 4$

Finally, substitute and solve...

$2 \times 4 + 3y = 11$, so $y = 1$

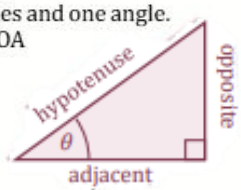
Right angled triangles G20, G22

Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA

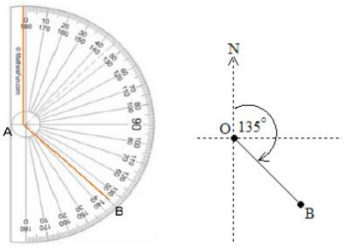


$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Use "2ndF" or "SHIFT" key to find a missing angle

Bearings

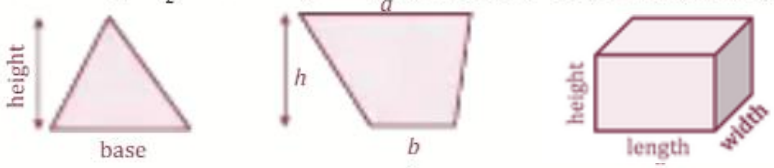
1. Measure from the North line.
2. Always need 3 digits (add a 0 in front if there are only 2)
3. Always measure clockwise.



Areas and volumes G16, G17, G18, G23

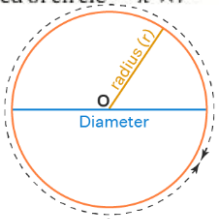
Area of triangle $= \frac{1}{2} \times$ base $\times$ height

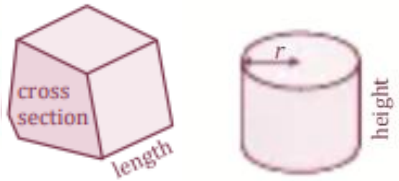
Volume of cuboid = length $\times$ width $\times$ height



Area of trapezium $= \frac{1}{2}(a+b) \times h$

Circumference of circle $= \pi \times D$
Area of circle $= \pi \times r^2$





Volume of cylinder $= \pi r^2 \times$ height

Volume of prism = area of cross section $\times$ length

Transformations G7, G8

Reflection
- Line of reflection

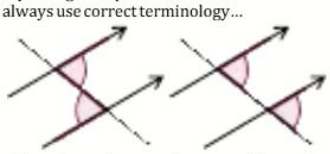
Translation
- Vector

Rotation
- Centre of rotation
- Angle of rotation
- Clockwise or anticlockwise

Enlargement
- Centre of enlargement
- Scale factor (if SF < 1 the shape will get smaller).

Angle facts

Equal angles in parallel lines: always use correct terminology...



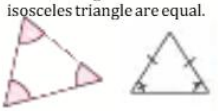
Alternate angles Corresponding angles

Co-interior angles add to 180°

Angles on a straight line total 180°

Angles in a full turn total 360°

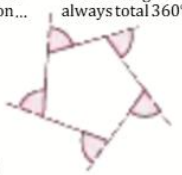
Interior angles in a triangle total 180°
The base angles in an isosceles triangle are equal.



Use this for the interior angles of any polygon...

...or $180° \times (n-2)$

Exterior angles always total 360°



What revision resources can help you revise?

Website	Log-in details	QR Codes
MathsWatch	Username: (firstname)(Lastname)@dustonschool Password: berrywood	
methodmaths	Centre ID: Duston Username: (firstname)(lastname) Password: berrywood	
Maths Genie	Non-required Contains adverts.	

Homework

Due date:	Set on:	Title