Year 11 Higher Maths Knowledge Organiser Term 1

Name:	Class:

Topics This term:

- Probability – GCSE Mathematics
  - Venn Diagrams
  - Relative Frequency
  - Tree Diagrams
- Time Series – GCSE Statistics
  - Trend Lines
  - Moving Averages
  - Seasonal Variation
- Number and Algebra – GCSE Mathematics
  - Re-cap and strengthening the Number and Algebra Skills

RESPECT

In Mathematics, a classroom environment should always be respectful. Students can show respect through:

- **Supporting each other with their learning.** Pupils should recognise that every individual has their own strengths and weaknesses and, as a class, we should 'up-lift' students.
- **Students should not be felt to be rushed by others in the classroom**. Respect that all students have different experiences and therefore will access the knowledge at different rates.
- **Being Polite.** As no different to the rest of school. Students should embrace diversity and treat all others with tolerance and decency.

ASPIRATION

- **Building logical processes.** Understanding that learning mathematical concepts improves our logical reasoning which improves other aspects of our lives: language, culture, games etc. the essence of mathematics is in respect of ideas, structures and relationships by logical reasoning.
- **Everyday needs.** Understanding that being numerate, along with literate, is a strong indicator of long-term success and students' ability to climb the tree of knowledge.

RESILIENCE

- **I don't know it… yet.** Understanding that maths can be abstract and that, as with anything new, it will take time to learn. With time, you will succeed.
- **Mathematical concept won't always come easily.** Understanding that getting things wrong is a frustrating and not pleasant feeling but, to succeed, it is a passage we need to go through.
- **Practice makes permanent.** Mathematics is a logical subject such that, rehearsal and repetition of method is the key to being successful and committing the knowledge to long-term memory. This process takes time and will come with failures along the way which we must persevere through.

Higher Tier Formulae Sheet GCSE Mathematics

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium = $\frac{1}{2}(a + b)\,h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle = $2\pi r = \pi d$

Area of a circle = πr^2

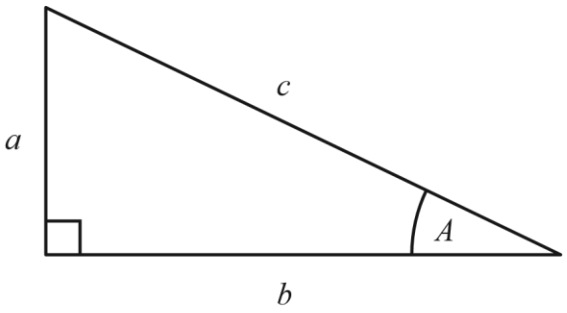
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

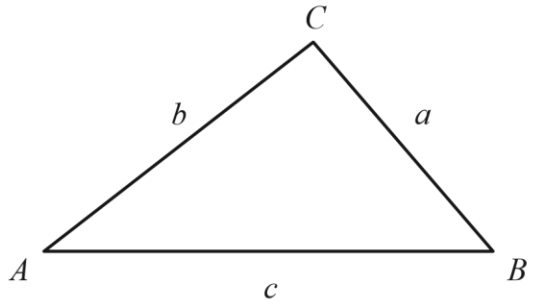


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$a^2 + b^2 = c^2$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}\,a\,b \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued = $P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

END OF EXAM AID

MathsWatch

Grade 4	
Clip	Topic
131	Index Notation
132	Introduction to Bounds
133	Midpoint of a Line on a Graph
134a	Expanding and Simplifying Brackets - Single
134b	Expanding and Simplifying Brackets - Double
135a	Solving Equations - Balancing
135b	Solving Equations - Float & Ping
136	Rearranging Simple Formulae
137	Forming Formulae and Equations
138	Inequalities on a Number Line
139	Solve Linear Inequalities
140	Simultaneous Equations Graphically
141	Fibonacci Sequences
142	Compound Units
143	Distance-Time Graphs
144	Similar Shapes
145a	Constructing Perpendiculars - Bisecting a Line
145b	Constructing Perpendiculars - From any Point
145c	Bisecting an Angle
146	Loci
147	Draw a Triangle Using Compasses
148	Enlargements
149	Tangents, Arcs, Sectors and Segments
150a	Pythagoras' Theorem - A Simple Approach
150b	Pythagoras' Theorem - An Algebraic Approach
150c	Pythagoras' Theorem - Line on a Graph
151	Simple Tree Diagrams
152	Sampling Populations
153	Time Series

Grade 5	
Clips	Topic
154	Negative Indices
155	Error Intervals
156	Mathematical Reasoning
157	Factorising and Solving Quadratics
158	The Difference of Two Squares
159a	Equation of a Straight Line - y=mx+c
159b	Equation of a Straight Line - Gradient
160	Roots and Turning Points of Quadratics
161	Cubic and Reciprocal Graphs
162	Simultaneous Equations Algebraically
163	Geometric Progressions
164	Compound Interest and Depreciation
165a	Ratio Questions - Standard Questions
165b	Ratio Questions - Questions with Overlap
165c	Ratio Questions - Ratios, Fractions, Equations
166	Congruent triangles
167	Sectors of a Circle
168	Trigonometry
169	Spheres
170	Pyramids
171	Cones
172	Frustums
173	Exact Trigonometric Values
174	Introduction to Vectors
175	Harder Tree Diagrams
176	Stratified sampling

Grade 6	
Clip	Topic
177	Recurring Decimals to Fractions
178	Product of Three Binomials
179	Iteration - Trial and Improvement
180	Iterative Processes
181a	Enlargement - Negative SF - Construction Lines
181b	Enlargement - Negative SF - Column Vectors
182	Combinations of Transformations
183	Circle Theorems
184	Proof of Circle Theorems
185	Probability using Venn Diagrams
186	Cumulative Frequency
187	Boxplots

Grade 7	
Clip	Topic
188	Fractional Indices
189	Recurring Decimals - Proof
190	Rearranging difficult Formulae
191	Solving Quadratics with the Formula
192	Factorising Hard Quadratics
193	Algebraic Proof
194	Exponential Functions
195a	Trigonometric Graphs - Sine and Cosine
195b	Trigonometric Graphs - Tangent
196a	Transformation of Functions - Polynomial
196b	Transformation of Functions - Trigonometric
197	Equation of a Circle
198	Regions
199	Direct and Inverse Proportion
200a	Ratio Questions - Standard Questions
200b	Ratio Questions - Ratios to Equations
200c	Ratio Questions - Equations to Ratios
201	Similarity - Area and Volume
202a	The Sine Rule
202b	The Cosine Rule
203	Area of a Triangle Using Sine
204	And and Or Probability Questions
205	Histograms

Grade 8 and 9	
Clip	Topic
206	Upper and Lower Bounds
207a	Surds - Introduction to Surds
207b	Surds - Surd Expressions
207c	Surds - Rationalising the Denominator
208	Perpendicular Lines
209a	Completing the Square - Basics
209b	Completing the Square - Solving
209c	Completing the Square - Sketching
210a	Algebraic Fractions - Simplifying
210b	Algebraic Fractions - Solving
211	Simultaneous Equations with a Quadratic
212	Solve Quadratic Inequalites
213	Finding the nth Term of a Quadratic
214a	Inverse Functions - Introduction
214b	Inverse Functions - Harder Questions
215	Composite Functions
216a	Interpreting Graphs - Velocity-Time Graphs
216b	Interpreting Graphs - Rate of Change
217	Pythagoras in 3D
218	Trigonometry in 3D
219	Vectors

Higher Tier Formulae

You must not write on this page.

Anything you write on this page will gain NO credit.

Skew = $\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$

Standard deviation = $\sqrt{\frac{1}{n}\sum(x-\bar{x})^2}$

An alternative formula for standard deviation is

standard deviation $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

Spearman's rank correlation coefficient $r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$

Rates of change (e.g. Crude birth rate = $\frac{number\ of\ births \times 1000}{total\ population}$)

END OF EXAM AID

GCSE Mathematics Higher Tier

Number · Algebra · Ratio, proportion and rates of change · Geometry & measures · Probability · Statistics

Listing strategies N5

Product rule for counting:
➔ $4 \times 3 \times 2 \times 1 = 24$ ways to arrange the letters P, I, X and L

Powers and roots N6, N7

Special indices: for any value a:

$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\left(\frac{p}{q}\right)} = \sqrt[q]{a^p}$$

➔ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

➔ $8^{\left(\frac{2}{3}\right)} = \sqrt[3]{8^2} = 4$

Surds N8

Look for the biggest square number factor of the number:
➔ $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

Rationalise the denominator N8

Multiply the numerator and denominator by an expression that makes the denominator an integer:

➔ $\frac{4}{\sqrt{7}} = \frac{4 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{4\sqrt{7}}{7}$

➔ $\frac{2}{4+\sqrt{5}}$

$= \frac{2}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{2(4-\sqrt{5})}{11}$

Standard form N9

Standard form numbers are of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Recurring decimals N10

Make a recurring decimal a fraction:
➔ $n = 0.23\dot{6}$
(two digits are in the recurring pattern, so multiply by 100)
$100n = 23.\dot{6}$
(this is the same as $23.6\dot{3}\dot{6}$)
$99n = 23.6\dot{3}\dot{6} - 0.23\dot{6} = 23.4$
$n = \frac{23.4}{99} = \frac{234}{990} = \frac{13}{55}$

Error intervals N15

Find the range of numbers that will round to a given value:
➔ $x = 5.83$ (2 decimal places)
$5.825 \le x < 5.835$
➔ $y = 46$ (2 significant figures)
$45.5 \le y < 46.5$
Note use of ≤ and <, and that the last significant figure of each is 5

Equations and identities A3

An equation is true for some particular value of x
➔ $2x + 1 = 7$ is true if $x = 3$
...but an identity is true for every value of x
➔ $(x+a)^2 \equiv x^2 + 2ax + a^2$
(note the use of the symbol ≡)

Laws of indices A4

For any value a:

$$a^x \times a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$

➔ $\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6}$ or $8q^9p^{-6}$

Difference of two squares A4

$$a^2 - b^2 = (a+b)(a-b)$$
➔ $x^2 - 25 = (x+5)(x-5)$

Rearrange a formula A5

The subject of a formula is the term on its own. Rearrange to
➔ Make x the subject of
$2x + ay = y - bx$
$2x + bx = y - ay$
$x(2+b) = y - ay$
$x = \frac{y-ay}{2+b}$

Functions A7

Combining functions:
$\mathrm{fg}(x) = \mathrm{f}(\mathrm{g}(x))$
➔ If $\mathrm{f}(x) = x + 3$ and $\mathrm{g}(x) = x^2$
$\mathrm{fg}(x) = x^2 + 3$
$\mathrm{gf}(x) = (x+3)^2$

The inverse of f is f^{-1}
➔ If $\mathrm{f}(x) = 2x + 5$ then
$\mathrm{f}^{-1}(x) = \frac{x-5}{2}$

$y = mx + c$ A9

Equation of straight line $y = mx + c$
m is the gradient; c is the y intercept:
➔ Find the equation of the line that joins (0 , 3) to (2 , 11)
Find its gradient...
$\frac{11-3}{2-0} = \frac{8}{2} = 4$
...and its y intercept...
Passes through (0 , 3), so $c = 3$
Equation is $y = 4x + 3$

Parallel lines: gradients are equal; perpendicular lines: gradients are "negative reciprocals".
➔ $y = 2x + 3$ and $y = 2x - 5$ are parallel to each other; $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$ are perpendicular

Transformations of curves A13

Starting with the curve $y = \mathrm{f}(x)$:
Translate $\begin{pmatrix}0\\a\end{pmatrix}$ for $y = \mathrm{f}(x) + a$
Translate $\begin{pmatrix}-a\\0\end{pmatrix}$ for $y = \mathrm{f}(x+a)$
Reflect in x axis for $y = -\mathrm{f}(x)$
Reflect y axis for $y = \mathrm{f}(-x)$

Velocity - time graph A15

Gradient = acceleration (you may need to draw a tangent to the curve at a point to find the gradient);
Area under curve = distance travelled.

Iteration A20

You will be given the formula to use:
➔ Solve $x^3 + 6x + 4 = 0$ by using the iteration $x_{n+1} = \sqrt[3]{6x_n - 4}$
Start with $x_1 = -2.8$
$x_2 = \sqrt[3]{6 \times (-2.8) - 4} = -2.750\ldots$
$x_3 = \sqrt[3]{6 \times (-2.750\ldots) - 4} = \cdots$
Repeat until you know the solution, or you do as many as the question says.

Quadratics A11, A18

If a quadratic equation cannot be factorised, use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

➔ Solve $2x^2 + 3x - 7 = 0$

$x = \frac{-3-\sqrt{9-(-56)}}{2\times2} = -2.73$

or $x = \frac{-3+\sqrt{9-(-56)}}{2\times2} = 1.23$

Complete the square to find the turning point of a quadratic graph.
➔ $y = x^2 - 6x + 2$
$y = (x-3)^2 - 9 + 2$
$y = (x-3)^2 - 7$
Turning point is at (3 , −7)

Equation of a circle A16

$x^2 + y^2 = r^2$ is a circle with centre (0 ,0) and radius r.
➔ $x^2 + y^2 = 25$ has centre (0 , 0) and radius 5

Simultaneous equations A19

One linear, one quadratic;
➔ Solve $\begin{cases} x + 3y = 10 \\ x^2 + y^2 = 20 \end{cases}$
Rearrange the linear, and substitute into the quadratic
$x = 10 - 3y$
so $(10-3y)^2 + y^2 = 20$
Expand and solve the quadratic
$100 - 60y + 9y^2 + y^2 = 20$
$10y^2 - 60y + 80 = 0$
$y = 2$ or $y = 4$
Finally, substitute into the linear and solve, pairing values...
$x + 3 \times 2 = 10$ so $(x , y) = (4 , 2)$
$x + 3 \times 4 = 10$ so $(x , y) = (-2 , 4)$

Sequences A24, A25

nth term of an arithmetic (linear) sequence is $bn + c$
➔ nth term of 5, 8, 11, 14, ... is $3n+2$ (always increases by 3 first term is $3 \times 1 + 2 = 5$)
nth term of a quadratic sequence is $an^2 + bn + c$
➔ First three terms of $n^2 + 3n - 1$ are 3, 9, 17, ...
Geometric sequence; multiply each term by a constant ratio
➔ 3, 6, 12, 24, ... (ratio is 2)
Fibonacci sequence; make the next term by adding the previous two ...
➔ 2, 4, 6, 10, 16, 26, 42, ...

Percentages: multipliers R9, R16

Percentage increase or decrease; use a multiplier (powers for repetition)
➔ Initially there were 20 000 fish in a lake. The number decreases by 15% each year. Estimate the number of fish after 6 years.
$20\,000 \times 0.85^6 = 7500$ (2sf)

Formula for compound interest
Total accrued $= P\left(1 + \frac{r}{100}\right)^n$
➔ I invest £600 at 3% compound interest. What is my account worth after 5 years?
$£600 \times \left(1 + \frac{3}{100}\right)^5 = £695.56$

Direct & inverse proportion R10

y is directly proportional to x:
$y = kx$ for a constant k
➔ b is directly proportional to a^2
$a = 6$ when $b = 90$ Find b if $a = 8$
$b = ka^2$ $a = 6$ and $b = 90$ for k
$90 = k \times 6^2$ so $k = 2.5$, $b = 2.5a^2$
$b = 2.5 \times 8^2 = 160$
y is inversely proportional to x
$yx = k$ or $y = \frac{k}{x}$ for a constant k

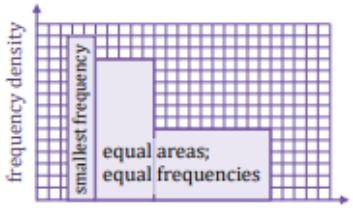
Probability rules P8, P9

Multiply for independent events
➔ P(6 on dice and H on coin)
$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
Add for mutually exclusive events
➔ P(5 or 6 on dice)
$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$
Apply these rules to tree diagrams.

In general...
P(A or B) = P(A) + P(B) − P(A and B)
P(A and B) = P(A given B) × P(B)

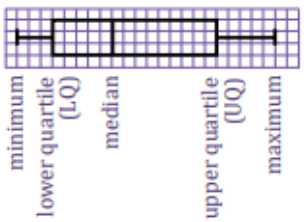
Histograms S3

Frequency = frequency density multiplied by class width. This means that bars with the same frequency have the same area.



Box plots S4

Interquartile range (IQR) = UQ − LQ



Here is pretty much all the Higher Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An ➔ points to an illustrative example. The codes refer to the DfE subject content. Pin this to a wall, keep it on your desk, carry it in your bag, make notes on it (sorry, don't take it into the examination)...

Standard graphs

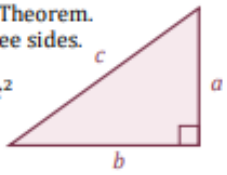
A12

$y = x^2$ $y = x^3$ $y = \frac{1}{x}$ $y = a^x$ $y = \sin(x°)$ $y = \cos(x°)$ $y = \tan(x°)$

Right angled triangles

G20

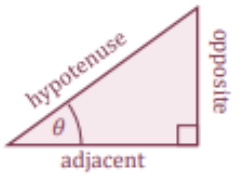
Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

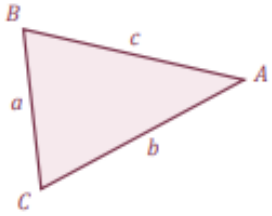
Use "2ndF" or "SHIFT" key to find a missing angle



The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Advanced trigonometry

G21, G22



A is opposite *a*
B is opposite *b*
C is opposite *c*

Sine Rule
Use if you are given an angle-side pair

Missing side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Cosine Rule
Use if you can't use the sine rule

Missing side: $a^2 = b^2 + c^2 - 2bc\cos A$

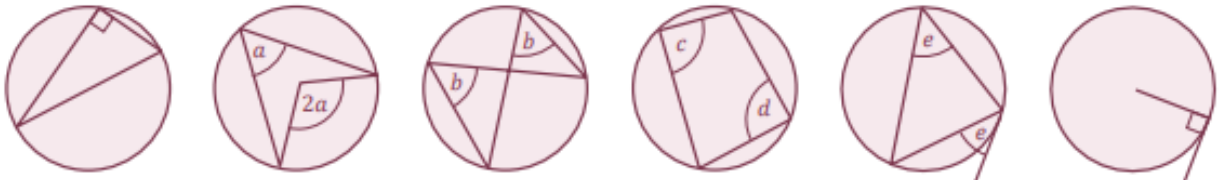
Missing angle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Special values of sin, cos, tan
Learn (or be able to find without a calculator)...

$\theta°$	$\sin\theta°$	$\cos\theta°$	$\tan\theta°$
0	0	1	1
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	

Circle theorems

G10



Angle in a semicircle is 90°

Angle at the centre is double the angle at the circumference

Angles in the same segment are equal

Opposite angles in a cyclic quadrilateral total 180°

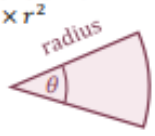
Alternate segment theorem

Tangent and radius are perpendicular

Areas and volumes

G16, G17, G18, G23

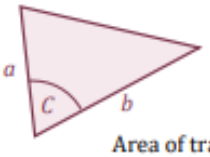
Circumference of circle = $\pi \times D$
Area of circle = $\pi \times r^2$

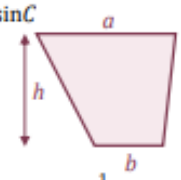


Arc length = $\frac{\theta}{360°} \times \pi \times D$

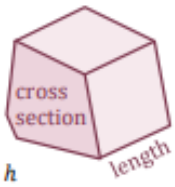
Area of sector = $\frac{\theta}{360°} \times \pi \times r^2$

Area of triangle = $\frac{1}{2}ab\sin C$

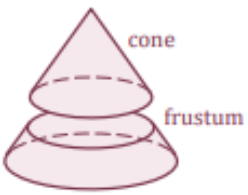




Area of trapezium = $\frac{1}{2}(a + b) \times h$



Volume of prism = area of cross section × length



Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of frustum is difference between the volumes of two cones

Transformations

G7, G8

Reflection
- Line of reflection

Translation
- Vector

Rotation
- Centre of rotation
- Angle of rotation
- Clockwise or anticlockwise

Enlargement
- Centre of enlargement
- Scale factor (if −1 < SF < 1 the shape will get smaller).

Similar shapes

G19

Ratios in similar shapes and solids:

• Length/perimeter	$1:n$	$a:b$
• Area	$1:n^2$	$a^2:b^2$
• Volume	$1:n^3$	$a^3:b^3$

What revision resources can help you revise?

Website	Log-in details	QR Codes
MathsWatch	Username: (firstname)(Lastname)@dustonschool Password: berrywood	
methodmaths	Centre ID: Duston Username: (firstname)(lastname) Password: berrywood	
Maths Genie	Non-required Contains adverts.	

Homework

Due date:	Set on:	Title